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Mathematics and arts

A CLASSIFICATION OF MATHEMATICAL SCULPTURE

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Abstract: *In this paper, we define the term* Mathematical Sculpture*, a task somehow complex. Also, we present a classification of mathematical sculptures as exhaustive and complete as possible. Our idea consists in establishing general groups for different branches of Mathematics, subdividing these groups according to the main mathematical concepts used in the sculpture design.*

Keywords: Mathematical sculpture

Introduction

There are several studies on the so-called *Mathematical Sculpture*, a concept that we will try to define in the next section.

These studies deal with specific aspects, such as the mathematical study of the works of a particular sculptor or the analysis of specific types of mathematical sculptures. Also, there are general studies. However, as far as we know, there is no work in the scientific literature providing a systematic analysis of the connections between mathematics and sculpture.

Neither is there any study that offers a complete and exhaustive classification of mathematical sculpture. The scarcity and lack of research on this artistic topic led us to choose it as the main topic of the doctoral thesis developed by Ricardo Zalaya, assistant lecturer at the Polytechnic University of Valencia, tutored by Javier Barrallo, professor at the University of the Basque Country, Spain.

In this paper we propose a classification for mathematical sculpture, based on the results of our research in the last years, and on the comments and observations provided by other experts on the topic. This approach was presented at the Alhambra ISAMA–BRIDGES 2003 Meeting [12].

Figure 1: Sculpture located in front of Picasso Tower, José María Cruz Novillo, Madrid, Spain, 1989.

At the meeting, the International Congresses of I.S.A.M.A (The International Society of the Arts, Mathematics and Architecture) and BRIDGES (society for the promotion of "bridges" between mathematics, arts and music) were held simultaneously [1]. These two associations include some experts in the field of mathematics and arts.

Mathematical sculpture works can be found in many places, in addition to museums and exhibition halls. Figure 1 shows a simple sculpture which presents an interesting geometric shape formed by thin cylindrical metal tubes. The design of this sculpture presents different concepts related to geometry and topology: surfaces (cylinders), intersections, symmetries, closed loops, etc. Geometry is the branch of mathematics more widely used in this sculpture.

Below, we have included two works of the well-known mathematical sculptor John Robinson, clearly illustrating the mental process of abstraction and subsequent geometrization (Figure 2). After, Figure 3 shows a more complex example by Bathsheba Grossman. Its design reflects several mathematical concepts: polyhedral geometry, surface topology, isometric transformations.

Most researchers and experts in mathematical sculpture come from United States of America, where this subject has gained great importance. Although in some West European countries, like Great Britain, and in other countries, like Japan, we can also find very good artists and experts on this topic. Among them is Carlo Sequin, professor at Berkeley University and a worldwide well-known expert. His webpage is an excellent reference [11].

Figure 2: Left: *Acrobats*, John Robinson, 1980; Right: *Elation*, John Robinson, 1983.

Figure 3: *Metatrino*, Bathsheba Grossman, 2007.

The Concept of Mathematical Sculpture

Before any attempt to classify the sculptures, we have to define the type of sculpture that we are trying to classify – the so-called *Mathematical Sculpture*. We propose the following definition:

Definition 1. *A* Mathematical Sculpture *is a sculpture that has mathematics as an essential element of conception, design, development or execution.*

In order to include a given artistic work in the set of sculptures that satisfy the definition, some mathematical concept or property must be significantly essential. In this definition we include from the simplest mathematical concept to the most complex mathematical concept (for instance, it may be trivial elementary geometry or sophisticated non-euclidean geometry). The definition is very general and covers a wide spectrum of possibilities, as one can understand by looking at the different artistic works analyzed here.

As an example, we present two sculptures based on the same concept – ruled surfaces, that is, surfaces that can be described as the set of points swept by a moving straight line in space. However, the complexity of both sculptures is

Figure 4: Left: *Ruled surface*, Andréu Alfaro, Valencia, Spain, 1982; Right: *Eclipse*, Charles O. Perry, Hiatt Regency Hotel, San Francisco, USA, 1973.

clearly different. The work shown on the left side of the Figure 4 is very simple whereas the sculpture shown on the right side of the Figure 4 is very complex. The latter sculpture is an extension of the concept of ruled surface, allowing the movement of any curve for the surface generation. It has been made by one of the most complete mathematical sculptors, Charles O. Perry [9, 10].

The mathematical sculptures included in our classification may use concepts related to many branches of mathematics: geometry, differential calculus or vector calculus, algebra, topology, logic, etc. An interesting example is the group of sculptures $\pi r^2 a$, made by Javier Carvajal (see [5], where the Spanish expert Eliseo Borrás summarizes his research on the mathematics used in the design of these Javier Carvajal's works).

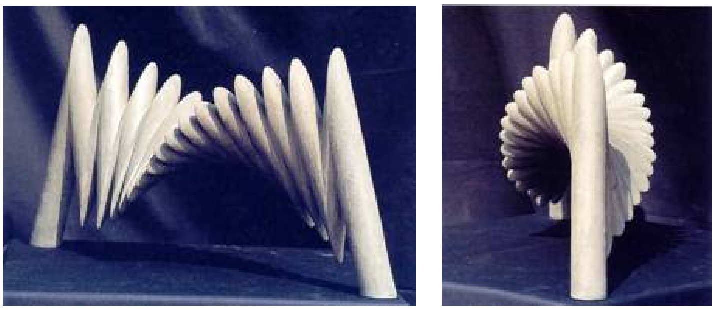

Figure 5: Left: *Parallel slices*, front view, Javier Carvajal; Right: *Parallel Slices*, side view, Javier Carvajal.

An element of this group is the sculpture presented in Figure 5. Another work by Javier Carvajal, from his series *Solomon Columns*, is presented in Figure 6. This example illustrates the difficulty of delimiting the concept of mathematical sculpture and whether a particular work may be considered as a mathematical sculpture.

Figure 6: *Two columns of inverse rotation*, series *Solominic Columns*, Javier Carvajal, 1991–1994.

Note that some sculptures explicitly show their mathematical nature; an example of that could be a polyhedron; however, in other works, the concepts are present in an implicit or hidden way, such as in the example of the series *Solominic Columns*.

To design the sculptures of the series *Solominic Columns*, the sculptor began by sectioning cylinders in order to obtain some initial objects. Each section is an ellipse and its position is characterized by the angle θ formed by the ellipse's major axis and the central axis of the cylinder (Figure 7, left). The plane determined by these two axes is the main plane of the section (Figure 7, blue plane on the left). Two different sections determine a module for the sculptures (Figure 7, center and right).

In addition to the respective angles, θ_1 and θ_2, the relative position of two sections is determined by ϕ, the angle formed by the two main planes, and by c, the distance between the centers of the ellipses. If c is large enough to prevent the intersection of the ellipses, the module is a "slic" (Figure 7, right). Otherwise, the two modules obtained are "segments" (Figure 7, center).

Each module, obtained using this procedure, is characterized by a 4-uple $(\theta_1, \theta_2, \phi, c)$, where $0° \leqslant \theta_1, \theta_2 \leqslant 90°$, $0° \leqslant \phi \leqslant 180°$, and c depends of the cylinder size. There is an infinite number of different possible modules.

It is possible to place one module beside another module with equal or different radius, rotating the modules by an angle α, and using direct or opposite orientations. Javier Carvajal used that idea to create pieces for his sculptures. Examples are ovoids, spheres, pumpkins, Solominic columns, torus, cones and swirling blades, etc. The translations and rotations of the modules generate shapes that frequently are similar to the geometrical figures found in nature.

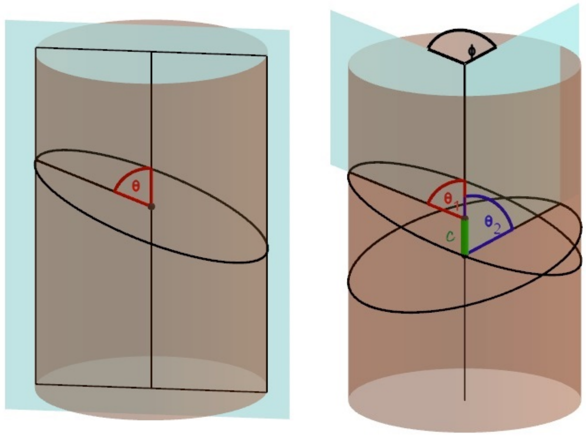

Figure 7: Sections of the pieces obtained from a cylinder and used by Javier Carvajal. Previous series of sculptures.

But not all shapes touch our senses in the same way. Some objects are more attractive than others and correspond to different numerical rhythms. In some works, the sculptor uses "polygonal spirals"; in another type of sculptures he uses "multipolygonal spirals" (Figure 8).

Figure 8: *Multipolygonal spirals*, series $\pi r^2 a$, Javier Carvajal, Spain, 1991–1994.

Since the main goal of this paper is the classification of mathematical sculpture, we don't use the standard mathematical notation. That is replaced by images and photos of sculptures and by grids and drafts used in their conception.

The use of the computer by many sculptors has allowed the development and evolution of mathematical sculpture. The computer has allowed the precise realization of very sophisticated sculptures. An example of this can be seen in Figure 9 (left), a sculpture by Bathsheba Grossman. Another example is a virtual piece made by the expert in computer science, Javier Barrallo, one of the authors of this paper. That is a case in which interdisciplinarity leads to interesting virtual experiences, such as this virtual sculpture from Barrallo's series *Hypersculpture*. Javier Barrallo uses parametric programming for the artistic design and fractal theory concepts for the work with textures [2, 3].


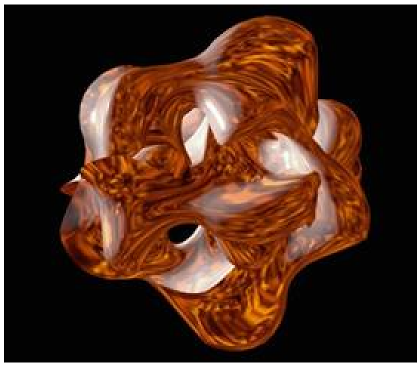

Figure 9: Left: *Seven spheres*, Bathsheba Grossman, 2005; Right: virtual sculpture of the series *Hypersculpture*, Javier Barrallo, 1994.

Educational purpose of classifying mathematical sculpture

A classification of mathematical sculpture provides a more systematic approach to this field, facilitating its incorporation in secondary or higher education. Courses devoted to connections between mathematics and arts already exist, being included in the contents of artistic and technical syllabi, such as in architecture.

We believe that, without a classification, courses devoted to mathematical sculpture lack structure, focusing only on the enumeration of a number of works or authors, based on particular studies. Examples of studies devoted to a particular sculptor can be consulted in [4, 9] (analyzis of the works of John Robinson and Charles O. Perry, respectively). Other examples can be found in [6, 7], written by the mathematical sculptor George Hart, about two particular types of his sculpture.

Other approaches for the classification of mathematical sculpture

The only approach to classify mathematical sculpture we know is based on the materials used, since they give the works varied geometrical properties. However, this typology does not permit to include all types of sculptures. The usual materials are the following:

- Wood. This material is used to emphasize curved surfaces (Figure 10, left). Due to its lightness, wood permits to create pieces that would otherwise be unstable (Figure 10, right).


Figure 10: Left: Photo of a Brent Collins workshop with several wood sculptures; Right: *Fire and ice*, George Hart, 1997.

- Welded metal. It is commonly used in polyhedral shapes (Figure 11).

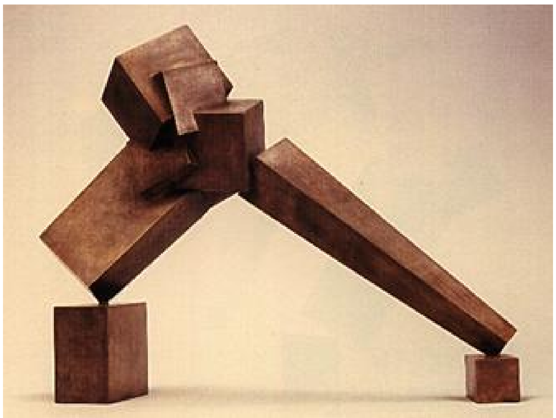

Figure 11: *Intersections II*, bronze, Bruce Beasley, 1991.

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- Concrete. Suitable for architectural works. A good example is the sculpture by Eduardo Chillida, shown in Figure 12.

Figure 12: *Elogio del horizonte*, concrete, Eduardo Chillida, Gijón, Spain, 1990.

- Stone. This material also emphasizes curved surfaces. As a result of the high weight, the works cannot be very complex. A good case is the sculpture by Nathaniel Friedman, shown in Figure 13. Nathaniel Friedman's works, made of stone, are examples of works difficult to classify as mathematical sculptures.

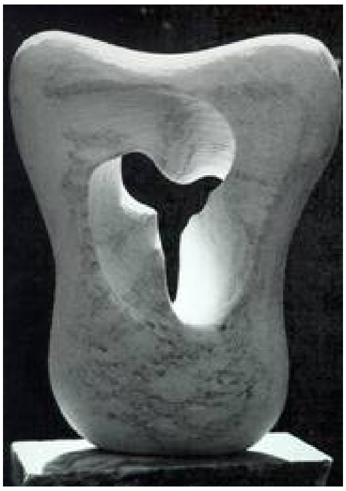

Figure 13: *Grand Canyon*, stone, Nathaniel Friedman, 1995.

The type of typology that we propose is based on mathematical properties. Our first approach was presented in 2003 (July), dividing the artistic works according to mathematical properties or concepts, or by combinations of both. The types considered were the following:

- Classic and polyhedral geometry
- Nonorientable surfaces
- Topological knots
- Quadric and ruled surfaces
- Modular and symmetric structures
- Boolean operations
- Minimal surfaces
- Transformations
- Others

As can be noted, some of the groups of this classification cover a wide range of sculptures. For example, "the classic and polyhedral geometry" group includes works with different properties. However, other groups included in this taxonomy, for example the "minimal surfaces" group, are more restricted.

A proposal for a classification of mathematical sculpture

Our first approach has been improved and we will present our final proposal in this section. This new classification is based on the different branches of mathematics. The limits between the different groups are not very strict, which is not surprising since the same thing happens regarding the limits of the different branches of mathematics. Our final proposal is the following:

- **Sculpture with geometric characteristics**
  - Polyhedrons
  - Curved mathematical surfaces
    - Quadric and surfaces of revolution
    - Ruled surfaces
    - Nonorientable surfaces
    - Minimal or zero-mean curved surfaces
  - Other surfaces
- **Sculpture with algebraic concepts**
  - Symmetry
  - Transformations and modular sculptures
  - Boolean operations
- **Topological sculpture**
- **Sculpture with varied mathematical concepts**

In some cases, the inclusion of a particular work in one of the groups may be difficult. A clear example of that difficulty is the sculpture by Bathsheba Grossman, shown in Figure 14. That work embraces several topics such as surfaces, topological knots, symmetry, etc. We will classify each particular case by attributing it a *dominating characteristic* that "dominates" its conception. That standard approach is explained in detail in [12].

Figure 14: *Alterknot*, Bathsheba Grossman, 1999.

It is important to note that the numbers of sculptures in each group have different magnitudes. There are groups with large sculpturesque potential, containing a large number of works. Some examples of that are the group type "Minimal or zero-mean curved surfaces" (an example is shown in Figure 15), and the group type "Nonorientable surfaces" (an example is shown in Figure 16).

Figure 15: *Minimal surface costa X*, snow, Helaman Ferguson, 1999.

Figure 16: *Nonorientable surface*, Brent Collins, 1985–1989.

General description and examples

In this section, we present a general description of the different groups mentioned in our classification, giving some examples to illustrate their main characteristics.

Geometrical sculpture

Geometrical sculpture is the widest group in the classification. That happens due to the intrinsic relation between plastic arts, specially sculpture, and geometry. Geometrical sculpture includes most of the mathematical sculpture. To check that fact, it is enough to look at the examples previously exposed, almost all examples included in this category — Figures 1, 2 (right), 4 (left and right), 5, 6, 8, 10 (left), and 14.

There are examples of sculptures for almost all possible types of solids, from the simplest ones like cubes, spheres, cones, cylinders, prisms, etc., to the most complex, like irregular polyhedrons or surfaces defined by highly complex mathematical equations. In addition, in some works, the most relevant element is not a particular type of solid or a combination of solids, but some property or properties.

Geometrical sculpture includes from simple shapes (an example is shown in Figure 17), to much more complex pieces (an example is shown in Figure 18). Also, regarding sizes, sculptures range from very small sculptures (Figure 14, with only 13 cm in height) to huge dimensions (Figure 4, right, with 13 m in height).

Figure 17: *Amaryllis (plant family)*, size 350 x 129 x 350 cm, Tony Smith, Wadswhorh Atheneum, Conneticut, USA, 1965.

Figure 18: *Escher on double torus*, size 15 cm, Helaman Ferguson.

Geometrical sculpture is a type of mathematical sculpture with a lot of tradition, specially in the 20th century. At the beginning of the century, cubist movement produced some works that can be included in this group. With respect to its origins, it is possible to look at [13], reference with analysis of important artistic trends and movements of the last century. Some artists, fans of abstract movements, minimal and conceptual movements, etc., made use of geometry.

Polyhedral sculpture is the first subcategory of Geometrical sculpture. First, we analyze the well-known platonic solids. These solids are some of the figures more widely used by mathematical sculptors and by other artists due to their beauty and simplicity. Although their description is well-known, it is worth mentioning some characteristics of these regular solids. A convex polyhedron is regular if all its faces are equal regular polygons and the same number of faces meet at every vertex. There are only five regular polyhedra, known as platonic (after the Greek philosopher Plato) or cosmic. These five solids are the following: tetrahedron (4 faces); hexahedron or cube (6 faces); octahedron (8 faces); icosahedron (20 faces) and dodecahedron (12 faces).

Like the platonic solids, the truncated polyhedrons[1] have been the inspiration for many mathematical sculptures. The number of possible cases is infinite. Archimedean solids are particular cases. An archimedean solid (or semiregular) is a convex polyhedron that has a similar arrangement of nonintersecting regular convex polygons of two or more different types arranged in the same way about each vertex with all sides the same length. Seven of the 13 Archimedean solids can be obtained by truncation of a platonic solid. These have also been widely used in sculpture.

Another type of figures commonly used by mathematical sculptors are those resulting from transformations of polyhedrons, such as deformation, star-shaping or rotation, or any other geometric transformation that may result in aesthetic effects. Figure 19 (left) shows a work by John Robinson, based on a dodecahedron. The faces have been replaced by 5-point stars. This work also presents other aesthetic values, like its color, or reflections depending on its illumination, etc.

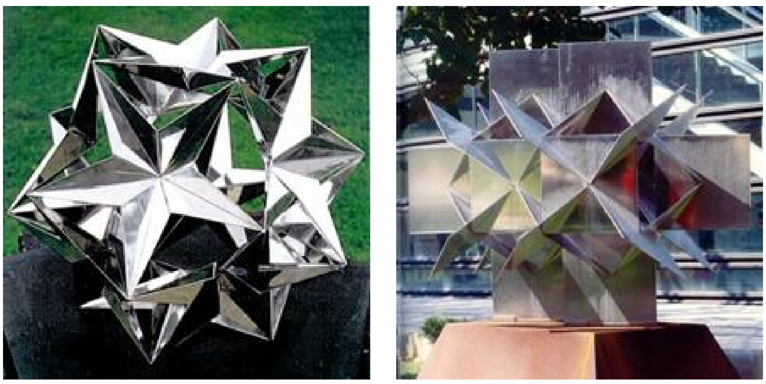

Figure 19: Left: *Star burst*, John Robinson, 1996. Right: *Permutational Sculpture*, Francisco Sobrino, Valencia, Spain, 2000.

Figure 19 (right) shows a work by Francisco Sobrino, consisting of a single module of stainless steel.

[1] Truncation is the removal of portions of solids falling outside a set of symmetrically placed planes.

Curved mathematical surfaces is the second subcategory of Geometrical sculpture. This subcategory has been subdivided into a few non-excluding types. For example, a surface widely used in both art and architecture is the hyperbolic paraboloid, also called saddle, which is simultaneously a quadric and a ruled surface. The further subcategories are the following.

Quadrics and surfaces of revolution: Quadrics are surfaces defined by a second degree polynomial equation (here, in three variables). Non-degenerated quadrics are: spheres, cones, cylinders, ellipsoids, hyperboloids (one or two sheets) and paraboloids (elliptic and hyperbolic). An example is shown in Figure 20.

Figure 20: *Hyperbolic paraboloid 8*, Jerry Sanders, 2000.

Surfaces of revolution, as the name suggests, are created by rotating a curve (the generatrix) around an axis of rotation. Surfaces of revolution have been used profusely in art and sculpture. An interesting simplification of human figures is shown in Figure 21.

Figure 21: *Couple*, Carmen Grau, Valencia, Spain, 2000.

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Ruled surfaces: Ruled surfaces are described as the set of points swept by a moving straight line in space. These surfaces have also inspired many artists and architects. An example is shown in Figure 22.

Figure 22: *Hyperbolic ribbed mace*, Charles O. Perry, Dublin Ohio, USA, 1987.

Nonorientable surfaces: A surface is orientable if a two-dimensional figure cannot be moved around the surface and back to where it started so that it looks like its own mirror image. Otherwise, the surface is nonorientable. The simplest nonorientable surface is the Möebius strip, one of the first objects of that kind that appeared in sculpture. A pioneer in mathematical sculpture, Max Bill, extensively used Möebius strips, obtaining very beautiful works, like the work shown in Figure 23 (see [8], an analysis of Max Bill's works, by Tom Marar).

Figure 23: *Endless surface*, Max Bill, Antverpen, Belgium, 1953–1956.

Another mathematical sculptor, Brent Collins, has developed many different models of nonorientable surfaces. One of his works can be seen in Figure 16. Also, the Japanese sculptor Keizo Ushio has based some of his works on the Möebius strip, or extensions of that concept, creating very simple, though splendid sculptures. Figure 24 shows a transformation of the double Möebius strip. That piece clearly illustrates the non-orientability of the surface.

Figure 24: *Mihama*, Keizo Ushio, 1990.

Minimal surfaces: Minimal surfaces, with zero-mean curvature, are surfaces of minimal surface area for given boundary conditions. A well-known example are the surfaces created by soap films. The sculptor Helaman Ferguson has created different works based on this concept, like the piece shown in Figure 15[2]. Another example, by Stewart Dickinson, is shown Figure 25.

Figure 25: *Ennepers minimal surface*, Stewart Dickinson.

[2]The Brazilian mathematician Celso Costa formulated its equations.

Other surfaces: This subcategory includes those surfaces that do not belong to any of the other types mentioned above. In this group, we have from sculptures with geometric objects as simple as planes, to others that can acquire very complex shapes or that combine different types of surfaces. Figure 26 shows a photo of a Richard Serra's exhibition. His works present simple geometrical shapes, planes, ellipses, truncated cones, etc. One of his most famous works, *Snake*, shown in the figure, is based on a third degree polynomial equation.

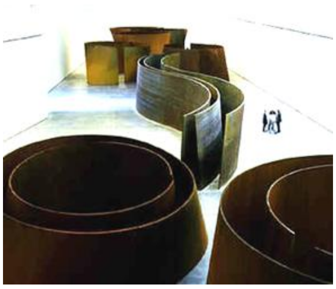

Figure 26: Some works by Richard Serra, exhibited in the Guggenheim Museum, Bilbao.

This subcategory also includes those surfaces given by equations not considered in the other groups. For example, transcendent equations such as trigonometric equations, exponential equations, etc.

Sculpture with algebraic concepts

This category comprises sculptures that make use of some algebraic concept. These works can also adopt geometric shapes like the sculptures of the first category. However, the algebraic properties that characterize them are so determinant for their conception, that we include them in this group.

Symmetry: One of the mathematical concepts with more occurrences in art is symmetry. Figure 27 illustrates a work by Robert Longhurst, showing symmetry with respect to the planes whose angles are multiple of sixty degrees.

Figure 27: *Arabesque XXIX*, Robert Longhurst, 2007.

Transformations and modular sculptures: The already mentioned works by Javier Carvajal fit the idea of transformation. The design of these sculptures is based on cylindric sections that subsequently are joined to create the complete pieces (Figures 5, 6 and 8). Modular sculptures are those sculptures in which a given pattern is repeated. The modules may be combined in many different ways. Brent Collins, a well-known mathematical sculptor, has created some modular sculptures, like those entitled *Modular spirals*. Figure 28 shows a modular sculpture by another artist, Michael Waren.

Figure 28: *Pascua*, Michael Waren, Valencia, Spain, 2000.

Boolean sculpture: In other works, operations with shapes are carried out, using some algebraic structure as, for example, boolean algebra. An example is the work by Bruce Beasley, already shown in Figure 11. The possible results of boolean operations are "true" or "false". This algebra applied to sculpture is used to describe how two solids relate, forming a new volume or emptiness. All logical operations are used: union, intersection, inversion, complement, and exclusion. Figure 29 (left and right) presents two crosses by Eduardo Chillida. The first one (left) can be interpreted as the complementary of the second (right), that is, its "negative".

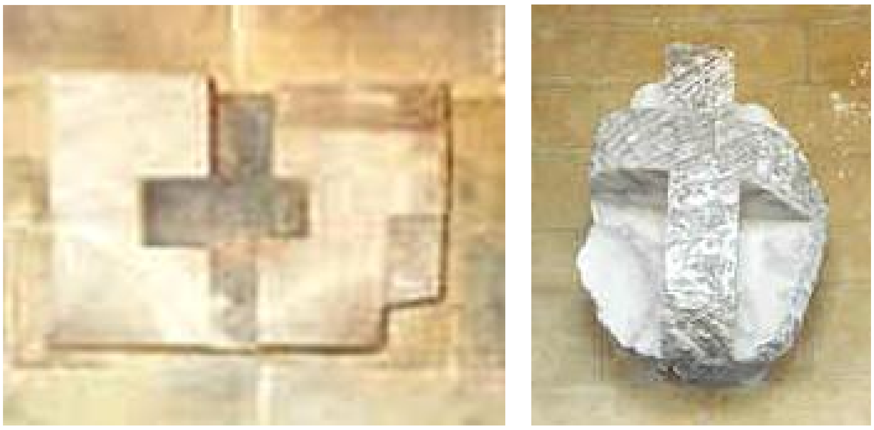

Figure 29: Left: cross in Santa Maria Church, Eduardo Chillida, San Sebastian, Spain, 1975; Right: cross in Buen Pastor Church, Eduardo Chillida, San Sebastian, Spain, 1997.

Topological sculpture

Mathematicians have studied "knots" for many centuries. This interesting and fascinating category of topological objects presents a wide range of possibilities to be used in sculpture. Most mathematical sculptors have made use of this concept. The examples in Figures 3, 14, and 18 belong to this group. Figure 30 (left) shows a sculpture by Keizo Ushio, a torus that, when sectioned by positioning a Möebius band, is divided into two topologically nested parts. Figure 30 (right) shows a computer image of the separation.

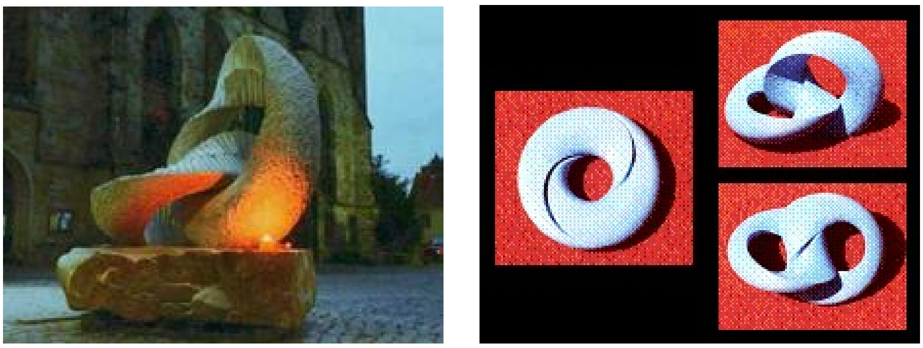

Figure 30: Left: *Oushi-Zokei* (sectioned torus), Keizo Ushio, San Sebastián, Spain, 1999; Right: separation.

Also, the mathematical sculptor John Robinson has made many works that can be included in this category. His works are simple, though very interesting from the topological point of view. Figure 31 shows his series *Trilogy*. These sculptures are inspired by the Borromean rings, three topological circles which are linked and form a Brunnian link (i.e., removing any ring results in two unlinked rings). The name of Borromean rings comes from their use in the coat of arms of the aristocratic Borromeo italian family.

Figure 31: Left: *Creation*, *Trilogy* series, John Robinson, 1990; Center: *Intuition*, *Trilogy* series, John Robinson, 1993; Right: *Genesis*, *Trilogy* series, John Robinson, 1995.

Sculpture with varied mathematical concepts

Although we develop and improve our classification, it is very difficult to include all the mathematical sculptures in the proposed categories. Because of that, we have established this last category. For example, the piece illustrated in Figure 31, by the sculptor Ken Herrick, has very little to do with those we have shown previously throughout the article.

Figure 32: *Cloud*, Ken Herrick.

Other interesting examples are some of Helaman Ferguson's sculptures, like the one shown in Figure 33. Although it is a nonorientable surface, we also highlight the texture, whose design required computer help. It is the Hilbert curve, a continuous fractal space-filling curve, first described by the German mathematician David Hilbert in 1891.

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Figure 33: *Umbilic Torus NC*, Helaman Ferguson, 1988.

In addition, Figure 34 shows a cube divided into two complementary fractal parts. We observe that, if we ignore Pauli's exclusion principle, when joining again the two parts, we obtain the original cube.

Figure 34: *The Unit Cube*, Bathsheba Grossman, 2002.

We must mention the possibilities for mathematical sculpture that can be open by the use of non-euclidean, elliptic and hyperbolic geometries. The sculptures motivated by these geometries should be included in this last category. We believe that the use of this type of geometries will occur more often in mathematical sculpture, as it happened in painting, especially after M. C. Escher legacy. An interesting example is shown in Figure 35.

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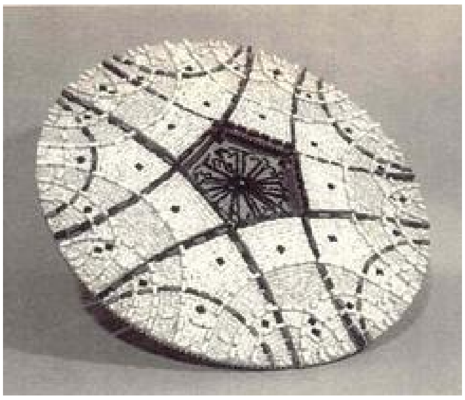

Figure 35: *Hyperbolic Diminution I*, Irene Rousseau, 2005.

Conclusions

- Mathematics relates to sculpture. Moreover, mathematics relates to most artistic manifestations.
- The breakthroughs in mathematics that have taken place in the 20th century have made possible the development of a new type of mathematical art.
- Mathematical sculpture has reached a remarkable status at present. To this has contributed, in addition to the recent advances in mathematics, the development of computer science.
- We believe that mathematical sculpture will expand. This is due to the causes mentioned above, as well as the growing interest of artists and public.
- Courses on mathematics and art, either at secondary level or at the university level, should be encouraged.
- Possibly, the proposed classification can be improved. Other concepts and different mathematical properties may be introduced.

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