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## Lewis Carroll in Numberland

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# *Articles*

## Lewis Carroll in Numberland

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**Abstract:** *Charles Dodgson is best known for his "Alice" books, "Alice's Adventures in Wonderland" and "Through the Looking-Glass", written under his pen-name of Lewis Carroll.*

*If he hadn't written them, he'd be mainly remembered as a pioneering photographer, one of the first to consider photography as an art rather than as simply a means of recording images. But if Dodgson had not written the Alice books or been a photographer, he might be remembered as a mathematician, the career he held as a lecturer at Christ Church in Oxford University.*

*But what mathematics did he do? How good a mathematician was he? How influential was his work?*

*In this illustrated paper, I'll describe his work in geometry, algebra, logic and the mathematics of voting, in the context of his other activities and, on the lighter side, I present some of the puzzles and paradoxes that he delighted in showing to his child-friends and contemporaries.*

**Key-words:** Lewis Carroll, *Alice's Adventures in Wonderland and Through the Looking-Glass.*

## 1 Introduction

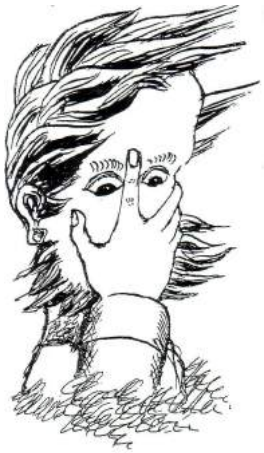
Letter written in verse to Margaret Cunnynghame from Christ Church, Oxford, on 30 January 1868.

*Dear Maggie,*
*No carte has yet been done of me*
*that does real justice to my smile;*
*and so I hardly like, you see,*
*to send you one. Meanwhile,*
*I send you a little thing*
*to give you an idea of what I look like*

*when I'm lecturing.*
*The merest sketch, you will allow -*
*yet I still think there's something grand*
*in the expression of the brow*
*and in the action of the hand.*

*Your affectionate friend, C. L. Dodgson*

*P. S. My best love to yourself, to your Mother my kindest regards,*
*to your small, fat, impertinent, ignorant brother my hatred. I think that is all.*



This letter to Margaret Cunnynghame shows up two aspects of Lewis Carroll - or the Revd. Charles Dodgson (his real name): his love of children and the fact that he was a teacher - in fact, a teacher of mathematics. If he hadn't written the Alice books, he would be mainly remembered as a pioneer Victorian photographer. And if he hadn't been known for that, he'd have been largely forgotten, as an Oxford mathematician who contributed very little. But is that really the case? What mathematics did he do?

Certainly, mathematics pervaded his life and works - even his Alice books (*Alice's Adventures in Wonderland and Through the Looking-Glass*) abound with mathematical language. For example, in the Mock Turtle scene, the Mock Turtle started:

*We went to school in the sea.*
*The master was an old turtle - we used to call him Tortoise.*
*Why did you call him tortoise if he wasn't one?*
*We called him tortoise because he taught us.*
*I only took the regular course. Reeling and writhing, of course, to begin with.*
*And then the different branches of arithmetic - ambition, distraction,*
*uglification and derision. And how many hours a day did you do lessons?*
*Ten hours the first day, nine hours the next, and so on.*

*What a curious plan!*
*That's the reason they're called lessons - because they lessen from day to day.*

And in *Through the Looking-Glass*, the White Queen and the Red Queen set Alice a test to see whether she should become a queen.

*Can you do Addition? What's one and one and one and one and one and one and one and one and one and one?*
*I don't know. I lost count.*
*She can't do Addition. Can you do subtraction? Take nine from eight.*
*Nine from eight I can't, you know; but -*
*She can't do subtraction. Can you do division? Divide a loaf by a knife. What's the answer to that? Bread-and-butter, of course.*
*She can't do sums a bit!*

Mathematical ideas also appear in his other children's books. In *The Hunting of the Snark*, the Butcher tries to convince the Beaver that 2 plus 1 is 3:

*Two added to one - if that could be done,*
*It said, with one's fingers and thumbs!*
*Recollecting with tears how, in earlier years,*
*It had taken no pains with its sums.*
*Taking Three as the subject to reason about -*
*A convenient number to state -*
*We add Seven, and Ten, and then multiply out*
*By One Thousand diminished by Eight.*
*The result we proceed to divide, as you see,*
*By Nine Hundred and Ninety and Two:*
*Then subtract Seventeen, and the answer must be*
*Exactly and perfectly true.*

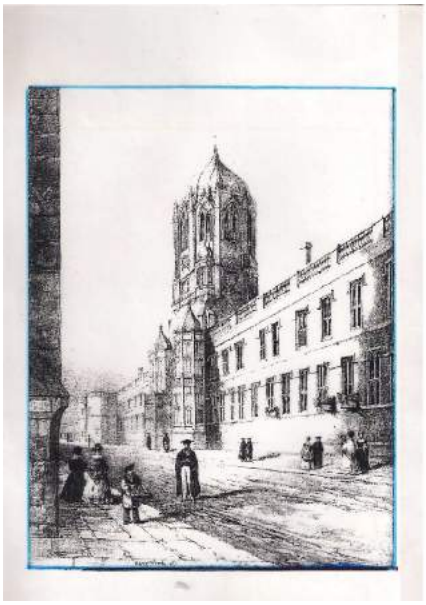
And in his last major novel, *Sylvie and Bruno concluded*, Dodgson's ability to illustrate mathematical ideas in a painless and picturesque way is used in the construction of *Fortunatus's purse* from three handkerchiefs. This purse has the form of a projective plane, with no inside or outside, and so contains all the fortune of the world. Since it cannot exist in three dimensions, he ceases just before the task becomes impossible.

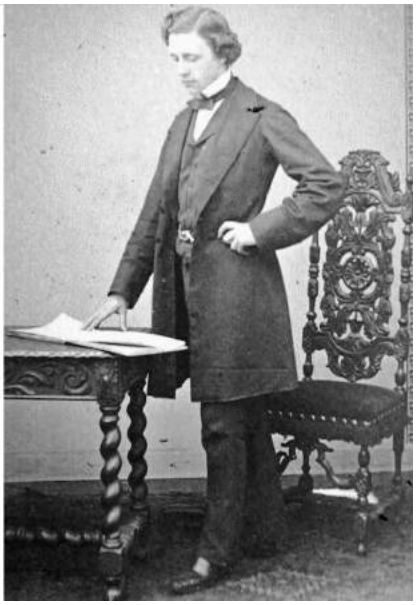
Another quirky result concerned map-making.

*There's another thing we've learned from your Nation - map-making. But we've carried it much further than you.*
*What do you consider the largest map that would be really useful?*
*About six inches to the mile.*
*Only six inches! We very soon got to six yards to the mile. Then we tried a hundred yards to the mile. And then came the grandest idea of all!*
*We actually made a map of the country, on the scale of a mile to the mile!*
*It has never been spread out, yet. The farmers objected: they said it would cover the whole country and shut out the sunlight! So we now use the country itself, as its own map, and I assure you it does nearly as well.*

## 2 Early Life

Charles Dodgson was born in 1832 into a "good English church family" in Daresbury in Cheshire, where his father, the Reverend Charles Dodgson, was the incumbent until 1843, when they all moved to Croft Rectory in Yorkshire. There he and his seven sisters and three brothers enjoyed a very happy childhood. When he was 14 he was sent to Rugby School, where he delighted in mathematics and the classics, but was never happy with all the rough-and-tumble.





In 1850 he was accepted at Oxford, and went up in January 1851 to Christ Church, the largest college, where he was to spend the rest of his life. His University course consisted mainly of mathematics and the classics, and involved three main examinations, starting in summer 1851 with his Responsions exams.

Letter written in old English from Christ Church, June 1851:

*My beloved and thrice-respected sister,*
*Onne moone his daye nexte we goe yn forre Responsions,*
*and I amme uppe toe mine*
*eyes yn worke. Thine truly, Charles.*

The next year he took his second Oxford examination - Moderations - gaining a First Class in Mathematics.

*Whether I shall add to this any honours at Collections*
*I can't at present say, but*

*I should think it very unlikely, as I have only today*
*to get up the work in The Acts of the*
*Apostles,* 2 *Greek Plays, and the Satires of Horace*
*and I feel myself almost totally*
*unable to read at all: I am beginning to suffer*
*from the reaction of reading for Moderations.*
*I am getting quite tired of being congratulated on*
*various subjects: there seems to be no*
*end of it. If I had shot the Dean, I could hardly have had more said about it.*

In the Summer of 1854, shortly before his Mathematics Finals examinations he went on a reading party to Yorkshire with the Professor of Natural Philosophy, Bartholomew Price - everyone called him "Bat" Price because his lectures were way above the audience. He was immortalized later in the Hatter's song:

*Twinkle, twinkle, little bat, How I wonder what you're at ...*
*Up above the world you fly, Like a tea-tray in the sky,*

Dodgson's Finals examinations took place in December 1854, and ranged over all areas of mathematics. Here's a question from that year, in the paper on Geometry and Algebra:

*Compare the advantages of a decimal and of a duodecimal system of notation in reference to* (1) *commerce;* (2) *pure arithmetic; and shew by duodecimals that the area of a room whose length is* 29 *feet* $7\frac{1}{2}$ *inches, and breadth is* 33 *feet* $9\frac{1}{4}$ *inches, is* 704 *feet* $30\frac{3}{8}$ *inches.*

He obtained the top mathematical First of his year:

*I must also add (this is a very boastful letter) that I ought to get the Senior Scholarship next term. One thing more I will add, I find I am the next* 1*st class Math. student to Faussett so that I stand next for the Lectureship.*

Dodgson did not win the Senior Scholarship, but he was appointed to the Mathematics Lectureship at Christ Church by the new Dean, the Rev. Henry Liddell, who was appointed in 1855 and whose daughters Alice, Edith and Lorina would soon become friends of the young Oxford don. Dodgson became the College's Sub-librarian, years later moving into a sumptuous suite of rooms for which the eminent artist William De Morgan, son of the mathematician Augustus De Morgan, designed the tiles around his fireplace.

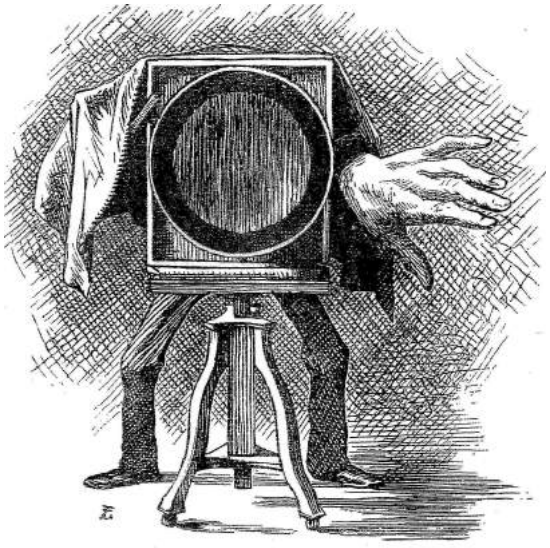
In his early years as a Christ Church lecturer Dodgson took up writing. His pen-name Lewis Carroll derived from his real name - Carroll (or Carolus) is the Latin for Charles, and Lewis is a form of Lutwidge, his middle name and mother's maiden name. He used it when writing for children, and in particular for his *Alice* books.

Around this time he also took up the hobby of photography, using the new wet collodion process. He was one of the first to regard photography as an art, rather than as simply a means of recording images, and if he were not known for

his Alice books, he would be primarily remembered as a pioneering photographer who took many hundreds of fine pictures - probably the greatest Victorian photographer of children. The Liddell daughters loved spending the afternoon with Mr Dodgson, watching him mix his chemicals, dressing up in costumes, and posing quite still for many seconds until the picture was done. A picture of Alice, dressed as a beggar girl, Alfred Tennyson described as the most beautiful photograph he had ever seen.

From *Hiawatha's photographing*:

*From his shoulder Hiawatha*
*Took the camera of rosewood*
*Made of sliding, folding rosewood;*
*Neatly put it all together.*
*In its case it lay compactly,*
*Folded into nearly nothing;*
*But he opened out the hinges,*
*Pushed and pulled the joints and hinges,*
*Till it looked all squares and oblongs,*
*Like a complicated figure*
*In the Second Book of Euclid ...*



## 3 Geometry

Mentioning Euclid brings us to Dodgson's enthusiasm for the writings of this great Greek author. Knowledge of Euclid's *Elements*, with its axiomatic structure and logical development, was required for all University candidates, as well as for the entrance examinations for the Army and the Civil Service, and dozens

of new editions appeared during Dodgson's lifetime.

To help his students, Dodgson produced a *Syllabus of Plane Algebraic Geometry*, described as the 'algebraic analogue' of Euclid's pure geometry, and systematically arranged with formal definitions, postulates and axioms. A few years later he gave an algebraic treatment of the Fifth book of Euclid's *Elements* - on proportion, and possibly due to Eudoxus - taking the propositions in turn and recasting them in algebraic notation.

But in geometry he's best known for his celebrated book *Euclid and his Modern Rivals*, which appeared in 1879. Some years earlier, the Association for the Improvement of Geometrical Teaching had been formed, with the express purpose of replacing Euclid in schools by newly devised geometry books. Dodgson was bitterly opposed to these aims, and his book, dedicated to the memory of Euclid, is a detailed attempt to compare Euclid's *Elements*, favourably in every case, with the well-known geometry texts of Legendre, J. M. Wilson, Benjamin Peirce, and others of the time. It is written as a drama in four acts, with four characters - Minos and Radamanthus (two of the judges in Hades, recast as Oxford dons), Herr Niemand (the phantasm of a German professor), and Euclid himself (who appears to Minos in a dream). After demolishing each rival book in turn, Euclid approaches Minos to compare notes, and to conclude that no other book should take the place of Euclid's *Elements*.

Dodgson's love of geometry surfaced in other places, too. His *Dynamics of a Parti-cle* was a witty pamphlet concerning the parliamentary election for the Oxford University seat. Dodgson started with his definitions, parodying those of Euclid:

Euclid: *A plane angle is the inclination of two straight lines to one another, which meet together, but which are not in the same direction.*

Dodgson: *Plain anger is the inclination of two voters to one another, who meet together, but whose views are not in the same direction.*

Euclid: *When a line, meeting another line, makes the angles on one side equal to those on the other, the angle on each side is called a right angle.*

Dodgson: *When a proctor, meeting another proctor, makes the votes on one side equal to those on the other, the feeling entertained by each side is called right anger.*

Euclid: *An obtuse angle is one which is greater than a right angle.*

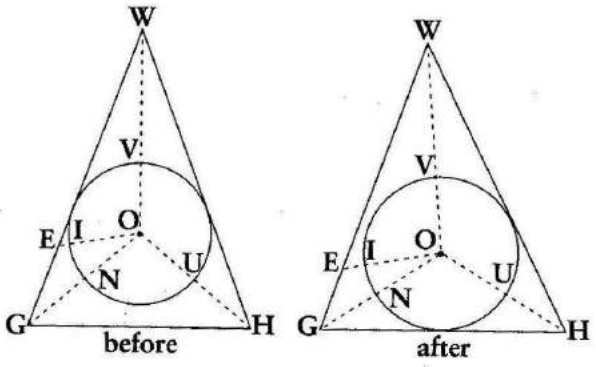
Dodgson: *Obtuse anger is that which is greater than right anger.*

He then introduced his postulates, which again parody those of Euclid:

1. *Let it be granted, that a speaker may digress from any one point to any other point.*
2. *That a finite argument (that is, one finished and disposed with) may be produced to any extent in subsequent debates.*
3. *That a controversy may be raised about any question, and at any distance from that question.*

And so he went on for several pages, leading up to a geometrical construction, where, WEG represents the sitting candidate William Ewart Gladstone (too liberal for Dodgson), GH is Gathorne-Hardy (Dodgson's preferred choice), and WH is William Heathcote, the third candidate.

*Let UNIV be a large circle, and take a triangle, two of whose sides WEG and WH are in contact with the circle, while GH, the base, is not in contact with it. It is required to destroy the contact of WEG and to bring GH into contact instead ... When this is effected, it will be found most convenient to project WEG to infinity.*




# 4 Algebra

Dodgson was an inveterate letter-writer - in the last 35 years of his life, he sent and received some 50000 letters, cataloguing them all. Although many letters were to his brothers and sisters or to distinguished figures of the time, the most interesting ones were to his child-friends, often containing poems, puzzles, and word-games. He had a deep understanding of their minds and an appreciation of their interests, qualities that stemmed from his own happy childhood experiences.

Most of his friendships were with young girls, such as with the Liddell children, but here's a letter to a young lad of 14, Wilton Rix.

*Honoured Sir,*
*Understanding you to be a distinguished algebraist (that is, distinguished from*

*other algebraists by different face, different height, etc.), I beg to submit to you a difficulty which distresses me much.*
*If* $x$ *and* $y$ *are each equal to* 1*, it is plain that*
$2(x^2 - y^2) = 0$*, and also that* $5(x - y) = 0$*.*
*Hence* $2(x^2 - y^2) = 5(x - y)$*.*
*Now divide each side of this equation by* $(x - y)$*. Then* $2(x + y) = 5$*.*
*But* $x + y = 1 + 1$*, i.e.* $x + y = 2$*. So that* $2 \times 2 = 5$*.*
*Ever since this painful fact has been forced upon me, I have not slept more than* 8 *hours a night, and have not been able to eat more than* 3 *meals a day.*
*I trust you will pity me and will kindly explain the difficulty to*

*Your obliged, Lewis Carroll.*

In 1865, Dodgson wrote his only algebra book, *An Elementary Treatise on Determinants*, with their *Application to Simultaneous Linear Equations and Algebraical Geometry*. In later years the story went around, which Dodgson firmly denied, that Queen Victoria had been utterly charmed by *Alice's Adventures in Wonderland* -"Send me the next book Mr Carroll produces" - the next book being the one on determinants - "We are not amused." Unfortunately, Dodgson's book didn't catch on, because of his cumbersome terminology and notation, but it did contain the first appearance in print of a well-known result involving the solutions of simultaneous linear equations. It also included a new method of his for evaluating large determinants in terms of small ones, a method that Bartholomew Price presented on his behalf to the Royal Society of London, who subsequently published it in their *Proceedings*.

## 5 The theory of voting

Another interest of his was the study of voting patterns. Some of his recommendations were adopted in England, such as the rule that allows no results to be announced until *all* the voting booths have closed. Others, such as his various methods of proportional representation, were not. As the philosopher Sir Michael Dummett later remarked:

*It is a matter for the deepest regret that Dodgson never completed the book he planned to write on this subject. Such was the lucidity of his exposition and mastery of this topic that it seems possible that, had he published it, the political history of Britain would have been significantly different.*

The simplest example that Dodgson gave of the failure of conventional methods is that of a simple majority.

| Electors | 1 - 3 | | | 4 - 7 | | | | 8 - 10 | | | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Ist Preference | a | a | a | b | b | b | b | c | c | c | d |
| 2nd preference | c | c | c | a | a | a | a | a | a | a | a |
| 3rd preference | d | d | d | c | c | c | c | d | d | d | c |
| 4th preference | b | b | b | d | d | d | d | b | b | b | b |

*There are eleven electors, each deciding among four candidates a, b, c, d.*
*The first three of the electors rank them a, c, d, b; the next four rank them b, a, c, d; and so on. Which candidate, overall, is the best?*
*Candidate a is considered best by three electors and second-best by the remaining eight electors. But in spite of this, candidate b is selected as the winner, although he is ranked worst by over half of the electors.*

Another interest of Dodgson's was the analysis of tennis tournaments:

*At a lawn tennis tournament where I chanced to be a spectator, the present method of assigning prizes was brought to my notice by the lamentations of one player who had been beaten early in the contest, and who had the mortification of seeing the second prize carried off by a player whom he knew to be quite inferior to himself.*

To illustrate Dodgson's irritation, let us take sixteen players, for example, ranked in order of merit, and let us organise a tournament with 1 playing 2, 3 playing 4, and so on. Then the winners of the first round are 1, 3, 5, ..., those of the second round are 1, 5, 9 and 13, and the final is between players 1 and 9, with player 1 winning first prize and with player 9 winning second prize even though he started in the lower half of the ranking. To avoid such difficulties, Dodgson managed to devise a method for re-scheduling all the rounds so that the first three prizes go to the best three players; this presaged the present system of seeding.

## 6 Puzzles

We now turn to more light-hearted pursuits - the puzzles he enjoyed showing to his young child-friends and to other adults. For more examples, see [4].
It was during his early years as a lecturer that he started to teach a class of young children at the school across the road. He varied the lessons with stories and puzzles, and may have been the first to use recreational mathematics as a vehicle for teaching mathematical ideas.

Here's one of his paradoxes, based on the well-known 1089 puzzle [1], but involving pounds, shillings and pence - remember that there are 12 pence in a shilling and 20 shillings in a pound.

*Put down any number of pounds not more than twelve, any number of shillings under twenty, and any number of pence under twelve. Under the pounds put the number of pence, under the shillings the number of shillings, and under the pence the number of pounds, thus reversing the line.*
*Subtract - reverse the line again - then add.*
*Answer, £12 18s. 11d., whatever numbers may have been selected.*

Another problem, hotly debated in Carroll's day, was the *Monkey on a rope puzzle*. A rope goes over a pulley - on one side is a monkey, and on the other is an equal weight. The monkey starts to climb the rope - what happens to the

weight? Some people thought that it went *up*, while others said that it went down.

A puzzle book of his, *A Tangled Tale*, contains ten stories each involving mathematical problems. Here's its preface - can you guess which child-friend he dedicated it to?

*Beloved Pupil! Tamed by thee,*
*Addish-, Subtrac-, Multiplica-tion,*
*Division, Fractions, Rule of Three,*
*Attest thy deft manipulation!*
*Then onward! Let the voice of Fame*
*From Age to Age repeat thy story,*
*Till thou hast won thyself a name*
*Exceeding even Euclid's glory!*

The second letters of each line spell *Edith Rix*, the sister of Wilton Rix to whom he wrote the algebra letter earlier.

# 7 Logic

Throughout his life, Mr Dodgson was interested in logic. In *Through the Looking-Glass*, Tweedledum and Tweedledee are bickering as always:

*I know what you're thinking about - but it isn't so, nohow.*
*Contrariwise - if it was so, it might be; and if it were so, it would be;*
*but as it isn't, it ain't. That's logic.*

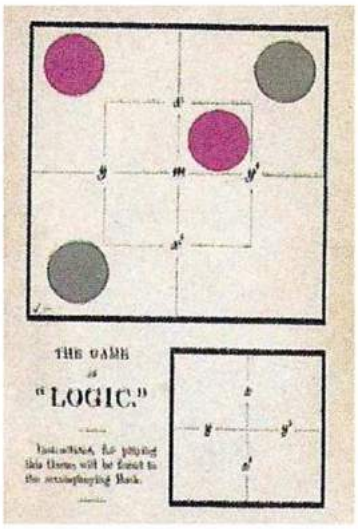
Dodgson believed that symbolic logic could be understood by his many child-friends, and devised *The Game of Logic* in order to help them sort out syllogisms. This contained a board and nine red and grey counters which are placed on sections of the board to represent true and false statements in order to sort out syllogisms.

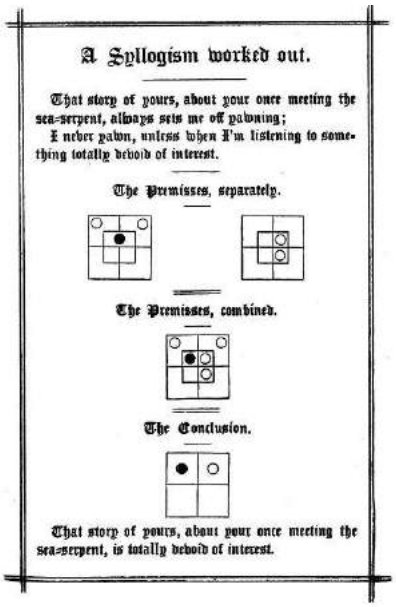
*That story of yours, about your once meeting the sea-serpent, always sets me off yawning. I never yawn, unless when I'm listening to something totally devoid of interest.*

Conclusion: *That story of yours, about your once meeting the sea-serpent, is totally devoid of interest.*

As he claimed:

*If, dear Reader, you will faithfully observe these Rules, and so give my book a really fair trial, I promise you most confidently that you will find Symbolic logic to be one of the most, if not the most, fascinating of mental recreations.*




In this first part he carefully avoided all difficulties which seemed to be beyond the grasp of an intelligent child of (say) twelve or fourteen years of age. He himself taught most of its contents to many children, and found them to take a real intelligent interest in the subject. Some are his examples are quite straightforward:

*Babies are illogical.*
*Nobody is despised who can manage a crocodile.*
*Illogical persons are despised.*
Conclusion: *Babies cannot manage crocodiles.*

Others needed more thought, but can readily be sorted out using his counters. The following example contains five statements, but the most ingenious of his examples went up to forty or more:

*No kitten that loves fish is unteachable.*
*No kitten without a tail will play with a gorilla.*
*Kittens with whiskers always love fish.*
*No teachable kitten has green eyes.*
*No kittens have tails unless they have whiskers.*
Conclusion: *No kitten with green eyes will play with a gorilla.*

Sadly, Dodgson died just before Volume 2 of his *Symbolic Logic* was completed, and his manuscript version did not turn up until the 1970s. If it had appeared earlier then Charles Dodgson might have been recognised as the greatest British logician of his time.

But let's leave the final word with Lewis Carroll. One night in 1857, while sit-

ting alone in his college room listening to the music from a Christ Church ball, he composed a double acrostic, one of whose lights has often been quoted as his own whimsical self-portrait:

*Yet what are all such gaieties to me*
*Whose thoughts are full of indices and surds?*
$x^2 + 7x + 53 = \frac{11}{3}$.

**Acknowledgements**: An earlier version of this article appeared in the *Transactions of the Manchester Statistical Society* (2011-12), 38-48.

For biographical information about Charles Dodgson, see [2]. For his photographic work, see [3]. For information about his life as a mathematician in Oxford, see [4].

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