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Patterns, mathematics and culture: The search for symmetry in Azorean sidewalks and traditional crafts

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Patterns, mathematics and culture: The search for symmetry in Azorean sidewalks and traditional crafts

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Abstract: *In the hustle and bustle of daily life, how often do we stop to pay attention to the tiny details around us, some of them right beneath our feet? Such is the case of interesting decorative patterns that can be found in squares and sidewalks beautified by the traditional Portuguese pavement. Its most common colors are the black and the white of the basalt and the limestone used; the result is a large variety and richness in patterns. No doubt, it is worth devoting some of our time enjoying the lovely Portuguese pavement, a true worldwide attraction. The interesting patterns found on the Azorean handicrafts are as fascinating and substantial from the cultural point of view. Patterns existing in the sidewalks and crafts can be studied from the mathematical point of view, thus allowing a thorough and rigorous cataloguing of such heritage. The mathematical classification is based on the concept of symmetry, a unifying principle of geometry. Symmetry is a unique tool for helping us relate things that at first glance may appear to have no common ground at all. By interlacing different fields of endeavor, the mathematical approach to sidewalks and crafts is particularly interesting, and an excellent source of inspiration for the development of highly motivated recreational activities. This text is an invitation to visit the nine islands of the Azores and to identify a wide range of patterns, namely rosettes and friezes, by getting to know different arts & crafts and sidewalks.*

Key words: Portuguese pavement, traditional Azorean arts & crafts, symmetry groups, rosettes, friezes.

1 Introduction

For being associated to the need of counting, calculating and organizing space and shapes, Mathematics is generally known as the science of quantity and space. Yet, this is a simplified and incomplete definition, particularly if one takes into account its extraordinary evolution over the last centuries and the many branches that have meanwhile been created. Nowadays, "Mathematics: the science of patterns" is the definition that is widely accepted by the academic community. The mathematician's work is, then, to find, study and classify all types of patterns. This sometimes-strenuous job helps to understand better the reality around us.

The nine islands of the Azores Archipelago are divided in three geographical groups, located in the Atlantic Ocean: the Eastern Group, comprising Santa Maria and S. Miguel, the Central Group, including Terceira, Graciosa, S. Jorge, Pico and Faial, and the Western Group, composed by Corvo and Flores. In the Azores, there are many sidewalks and squares beautified by the traditional Portuguese pavement. Due to a great diversity of materials and techniques employed, the quality of the local arts & crafts proves astonishing to whomever visits the islands.

By embracing the challenge to discover and classify the patterns that can be found in the Azorean Portuguese Pavement and in the Traditional Azorean Crafts, those interested in the subject can achieve a better understanding of the mathematician's concern with the organization of information by "shelves", according to certain previously defined criteria. They can also enjoy the feeling of harmony of proportions and the search for an order, inherent to the concept of symmetry. Therefore, this is an excellent topic for promoting interesting recreational mathematical activities, thus connecting Mathematics with Culture.

2 What is symmetry and how Mathematics measure it

To simplify the classification of patterns, we will treat the examples of decorative and ornamental art (belonging to the 3-dimensional world) as if they were sets of points of the plane (2-dimensional), in other words, *figures* of the plane.

In the chosen examples, there should be a basic motif repeating itself. We will not be interested in the configuration of the motif (it could be a star, a flower, a petal, an abstract drawing or anything else), but in how the repetition occurs. To simplify the mathematical analysis, we should disregard minor flaws or irregularities and consider only two colors: the color of the figure and the background color.

Let us remember some important concepts. A *symmetry* of a figure is an *isometry* of the plane (a way of moving the points of the plane by keeping the distances among them) that maps the figure back onto itself. For example, when we rotate a square 90 degrees around its center, we still get a square in exactly the same position as we started (Fig. 1). We say the square has a rotational symmetry (in this case, of 90 degrees).

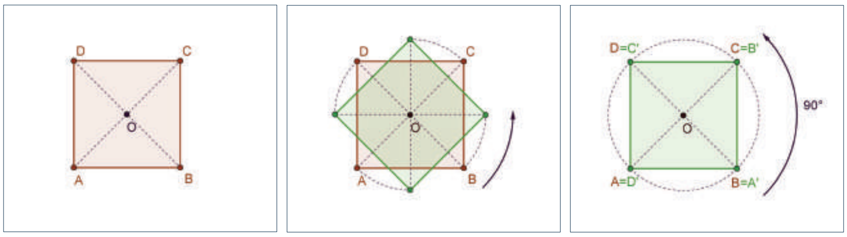

Figure 1: Rotational symmetry.

There are four types of symmetry:

- *reflection symmetry* or *mirror symmetry* (associated to a line, called the *axis of symmetry*);
- *rotational symmetry* (associated to a point, called the *rotation center*, and to a given amplitude);
- *translational symmetry* (associated to a vector, with a given direction and magnitude);
- *glide reflection symmetry* (resulting from the composition of a reflection with a translation of a vector parallel to the line that defines the reflection).

We can easily find these four types of symmetry in the Portuguese pavement. Some examples are given in Fig. 2.

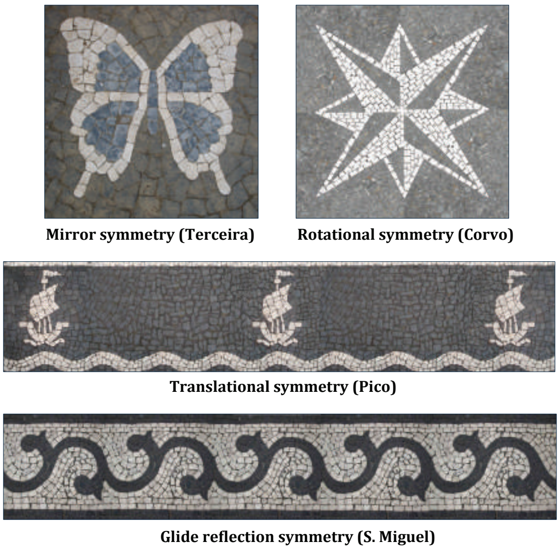

Figure 2: Four types of symmetry.

Next, it is shown in detail how a basic pavement can comprise the four types of symmetry (Fig. 3). Let's start with the best known type: the mirror symmetry. In (a), if we place a mirror perpendicularly to the plane of the figure, so that the edge of the mirror is set on the vertical red line, then we will see that each side of the image is indeed the reflection of the other (the same conclusion can be achieved by folding the paper that represents the plane along the red line). This line is called an axis of symmetry. Other axes of symmetry can be easily found as long as one bears in mind that the same pattern is repeated indefinitely to the right and to the left, beyond the photograph.

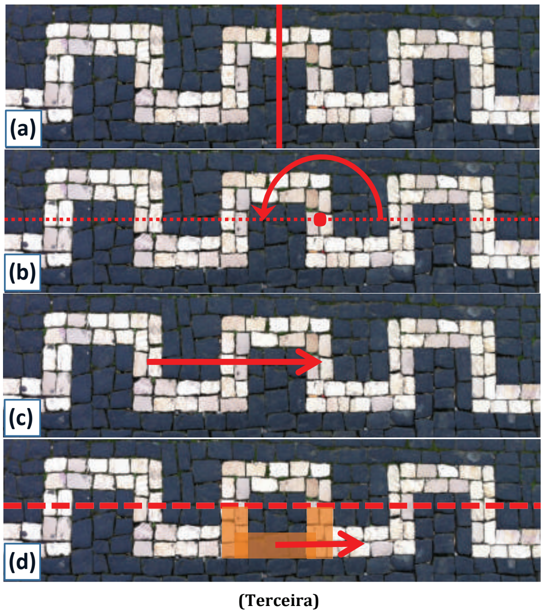

Figure 3: A survey.

There are also other types of symmetry apparently less perceptible. In (b) one can explore the concept of rotational symmetry. First, we have to choose an appropriate point: the rotation center. The idea is to rotate the figure around the fixed point according to an angle with a given amplitude. Generally, it's counterclockwise, or the positive direction, as it's called. If by rotating the figure according to an amplitude lower than 360 degrees it matches the initial position, then we say that it has rotational symmetry: the initial figure and the one resulting from the rotation occupy the exact same position; when the symmetry was applied to the plane the design moved onto itself. The reader can use a pencil and tracing paper to duplicate the outline of the pavement in (b). Then, overlap the image drawn to the original image, place the tip of a pen on the tracing paper, on the point marked in (b), and rotate the paper 180 degrees around that point (two right angles). You will reach the conclusion that the figure obtained overlaps entirely the original figure. This is a 180-degree rotational symmetry, also known as *half-turn*. If we bear in mind that the pattern repeats itself indefinitely and if we choose other appropriate rotation centers, we will find more half-turns.

There are two more types of symmetry to take into account. The translational symmetry is the symmetry that a figure has if it can be made to fit exactly onto the original when it is translated a given distance towards a given direction (according to a given vector). To illustrate that concept, use the sketch done on the tracing paper, overlap it to the figure in (c) and drag the tracing paper according to the direction and magnitude of the vector represented in (c). At the end of the process, you will reach the conclusion that there is a perfect overlapping of the two outlines.

Let's have a look at the last type of symmetry: the glide reflection symmetry. In (d), there is a dashed horizontal line. A closer look shows that this line is not an axis of symmetry of the figure. Nevertheless, if we apply the reflection in the given line, followed by a translational movement according to a vector parallel to the line with half the magnitude of the vector represented in (c), we quickly understand that the resulting figure overlaps the initial one[1].

For more details on isometries and symmetries, see [3, 5, 6, 8].

Now, let's see how to classify a figure based on its symmetries. The set of all symmetries of a figure forms a group under composition: the *symmetry group* of the figure. The classification of symmetry groups is summarized in Fig. 4 ([2]).

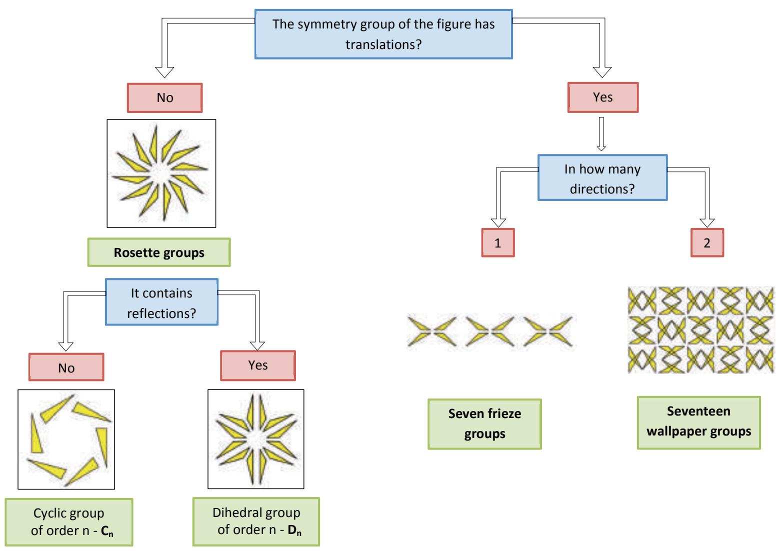

Figure 4: Symmetry groups.

[1]The exploration of these symmetries can be seen in: `https://youtu.be/aIgl9T658bk`.

Rosettes are figures with rotational symmetries and, in some cases, mirror symmetries. It can be proved [3] that only two situations can occur: their symmetry group is a *cyclic group* C_n (figures with n rotational symmetries) or a *dihedral group* D_n (figures with n rotational symmetries and n mirror symmetries). The rotational symmetries have all the same center and are associated to amplitudes of $360/n$ degrees and their multiple numbers. The axes of symmetry, when existing, all go through the rotation center.

Actually, you only have to identify the basic motif that is repeated around the rotation center and count the number of repetitions (n). Then all you have to do is check for rotational symmetries alone (C) or whether there are also mirror symmetries (D), as shown in Fig. 5.

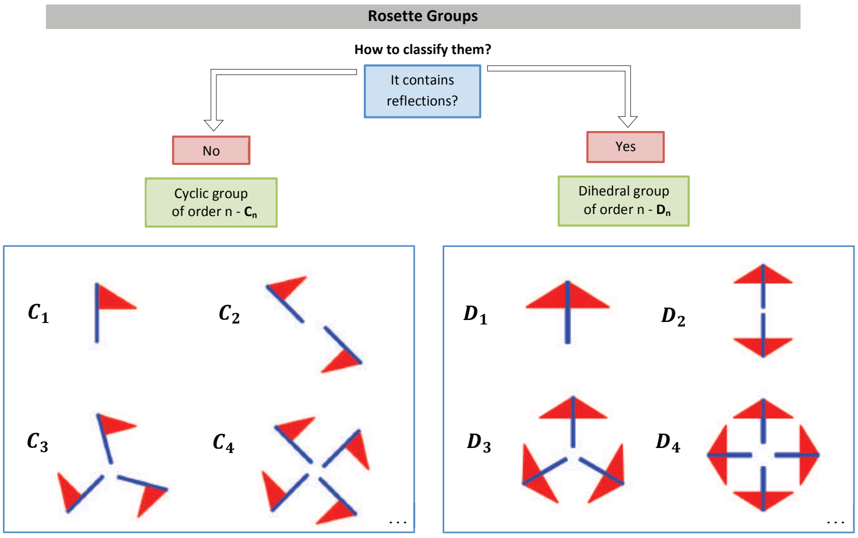

Figure 5: Rosettes.

A figure with symmetry group C_1 is called *asymmetric* (because it lacks symmetry), since the only way it can match the initial position is by the trivial rotation of $360/1 = 360$ degrees (or 0 degrees, if you prefer). A figure with symmetry group D_1, besides the trivial rotation, presents a mirror symmetry. For the symmetry group C_2, we have a rotational symmetry of $360/2 = 180$ degrees and another of $180 + 180 = 360$ degrees (that is, the trivial rotation). For group D_2 we also have to include two mirror symmetries (with perpendicular axes of symmetry). Furthermore, group C_3 comprises rotations of $360/3 = 120$ degrees, $120 + 120 = 240$ degrees and $120 + 120 + 120 = 360$ degrees. For group D_3, we have to add three mirror symmetries. And so forth and so on.

The *friezes* are figures that have translational symmetries in one single direction. Mathematics capacity to systematize the information has again prevalence here for it can be proved that there are only seven different ways of repeating a given basic motif along a strip by using the four types of symmetry [3].

In friezes, a rotational symmetry (if it exists) must be a 180 degrees one, also known as a half-turn. The reason for that is quite simple. As the motif is repeated across a plane following only one direction, using a rotation with an amplitude different from 180 degrees would subsequently displace the motif to a direction different from the one desired, i.e., outside the strip.

Another issue to take into account when classifying a frieze is its position. To avoid confusions, it's preferable to study it in a "horizontal position", that is, we should consider that the pattern repeats itself along a strip "parallel to the ground". Thus we can say without doubt that there is a *horizontal reflection* (when the axis of symmetry has the same direction of the strip) or a *vertical reflection* (when the axis of symmetry is perpendicular to the strip).

What are the differences between the seven frieze groups?[2] Fig. 6 presents a flowchart for the classification of the symmetry group of a frieze. We use the notation of Fejes Tóth [7]. The seven symmetry groups are represented by the letter F. When there is a half-turn a 2 is placed in the subscript position; otherwise a 1 is placed in that position. In the superscript position, it is placed a 1 (when there is a horizontal reflection), a 2 (when there is a vertical reflection) or a 3 (when there is a glide reflection). The absence of an exponent means that there are no mirror symmetries, as well as no glide reflection symmetries.

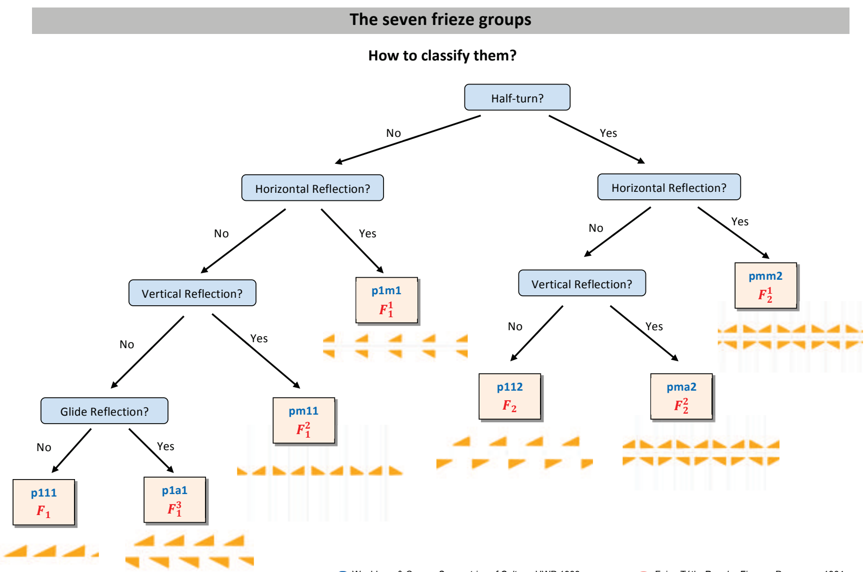

Figure 6: Flowchart for the classification of the friezes.

There are other notations. For instance, the four-symbol notation given in Fig. 6

[2]The exploration of the seven frieze groups using one domino can be seen in: `http://youtu.be/GKTNCNrOMhw`.

is the classical one introduced by Russian Crystallographers. It can be determined for a given frieze as follows: the first symbol, p, signals the existence of a translation; the second symbol is m if there is a vertical reflection and 1 otherwise; the third symbol is m if there is a horizontal reflection, a if there is a glide reflection and 1 otherwise; the fourth symbol is 2 if there is a half-turn and 1 otherwise [9].

If a figure is neither a rosette nor a frieze, then it must have translational symmetries in more than one direction (Fig. 4). The multiple translational directions force the pattern to cover the entire infinite plane. It can be proved that there are only 17 different ways of producing patterns under these criteria, which correspond to the 17 *wallpapers groups* [3].

Given the abundance of rosettes and friezes on Azorean sidewalks and Azorean traditional crafts, we propose a brief journey through the nine islands of the Azores to explore their symmetries.

3 Artistic crochet from Faial and Pico

In Faial and Pico, *needlework* or *artistic crochet* gives life to different decorative elements: flowers (the most characteristic element of the Azorean needlework, with interconnected rosettes representing the passion fruit flower, blackberries and grapes); geometrical (based on Greek motifs); and representations of everyday life in fine and delicate items either for domestic use or for decoration, as tablecloths, centerpieces, bed linen, doilies and other linen or cotton objects. This traditionally womanly craft had its highpoint in mid-twentieth century and is tightly connected to the emigration phenomena to the USA[3]. For more information, see [4].

Figure 7: Ana Baptista.

In the beginning of the 1950s, there were about five hundred full time needle

[3]The Regional Centre for Handicrafts's website: `http://www.artesanato.azores.gov.pt`.

lace workers. Nowadays that number has really decreased. What was once a fundamental means of subsistence for many families and the predominant activity for women, currently is a landmark of Azorean cultural heritage duly certified and deserving a place of honor in national and international exhibitions. The work of Ana Baptista (Fig. 7), of all the needle lace workers still active and still keeping this art alive, is worth mentioning for the perfection with which it is executed.

Ana Melo Baptista was born in the village of Flamengos, on the island of Faial, and currently lives in the city of Horta. She first began lace confection under the guidance of her older sister. Not yet out of school, she was already producing needle lace gloves. Stitches as the button hole and the eyelet were some of the most difficult challenges to overcome in the early years of apprenticeship of this young artisan, who spent all her available time, including evenings, learning. After some time, she was already her own boss and even employed several other workers, which serves to show how very interesting her professional profile is.

Let us have a look at some works made by this artisan (Fig. 8). Ana Baptista brings forth two pieces worth studying for their originality: a sunflower (a) and a rectangular doily with a different array of blackberries in the outer strip (b). The blackberries design has been complimented by many and orders for replicating the design on bed sheets and doilies for tea trays have been placed. The artisan has another interesting design: a doily with roses and beech leaves (c) which was chosen as the cover image of [4], a book on traditional lace.

Next, we will analyze the symmetries of some traditional lace pieces produced by Ana Baptista, whose promptitude and responsiveness we appreciate immensely.

We could analyze the sunflower, in (a), as a whole or we could focus on the flower in the center and on each of the following circular strips. We immediately find rotational symmetries: if we rotate the doily around its center according to a given amplitude, the figure obtained totally overlaps the initial figure. The amplitude to be used depends on the number of the motif's repetitions. For example, on the center flower there are 12 petals (12 repetitions), for which the rotation angle should have an amplitude of 360/12=30 degrees (or any of its multiples) as to obtain a symmetry of that flower. If we analyze the following circular strips of the doily, we find a strip with 24 "sunrays" (now the minimum amplitude is of $360/24 = 15$ degrees) and the outer strip has 18 sunflower petals (the minimum amplitude is of $360/18 = 20$ degrees).

In the situations analyzed, we can also find mirror symmetries (the number of axes of symmetry is equal to the number of repetitions of each motif). Such is the case of the flower in the center (D_{12}) and of the outer strip showing the sunflowers petals (D_{18}), but no longer the case of the "sunrays" strip (C_{24}). The use of a mirror allows the immediate conclusion that this strip does not have reflection symmetries. Thus, we are left with just rotational symmetries for the "sunrays" strip, which convey a feeling of movement around a point, as we get from looking at a weather vane or the sails of a windmill.

If we analyze the edge of the doily present in (b) and overlap the motif that

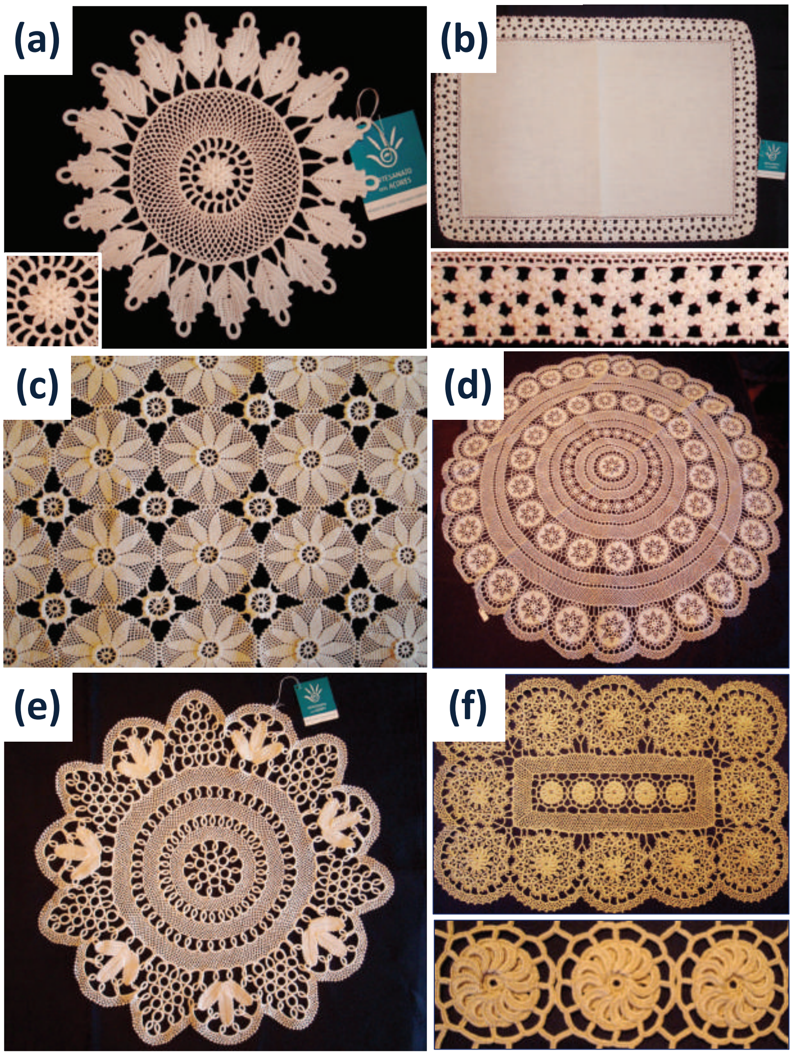

Figure 8: Works of Ana Baptista.

is repeated indefinitely from left to right, we get a frieze, that is, a figure with translational symmetries in only one direction. The strip in (b) has other types of symmetry as well: half-turn symmetries ($360/2 = 180$ degrees rotations; if we imagine the strip upside down, its configuration is not altered); vertical reflections (the axes of symmetry are perpendicular to the frieze); and glide reflections (following the same direction as the frieze, these symmetries produce an effect similar to our footprints when walking barefoot on the sand). Therefore, the symmetry group of the strip in (b) is F_2^2.

In (c) we can regard the flowers individually as rosettes. We can also consider strips of flowers, for example, horizontally or vertically and then get a frieze. If we analyze the doily as a whole, we get a 2-dimensional pattern (a wallpaper), for the flowers pave the whole plane.

Traditional lace, often called *Artistic crochet*, is an unending source of symmetries. Some more examples are put forth in (d), (e) and (f) and we dare the reader to find their symmetries!

4 Wheat Straw Embroidery from Faial

As if it were gold thread, wheat or rye straw is used by the embroiderers of Faial to decorate white or black tulle, creating unique evening gowns, bridesmaid dresses, scarves or doilies. The main decorative element is the wheat spike, although other vegetal or even figural elements are part of the motifs chosen by the embroiderers of the island[4]. For more information, see [1].

The uniqueness and splendor of the Azorean embroidery totally justifies its promotion on the regional, national and international levels. It is a genuinely Azorean product with quality and origin certification since 1998. On the island of Faial, the major embroideries are made of wheat straw on tulle.

We sat down and talked with Isaura Rodrigues, a well-known artisan famous for her wheat-straw-on-tulle expertise (Fig. 9).

Figure 9: Isaura Rodrigues.

An interesting fact pops up when talking about her life experience: this artisan is not just an expert on embroidery using wheat thread and tulle. Isaura rewinds the film of her life: “In 1998 I had to go live temporarily in a prefabricated unit. I missed the roomy house where I lived. I tried to overcome this less positive moment of my life by exploring various forms of artisanship. I started

[4]The Regional Centre for Handicrafts’s website: `http://www.artesanato.azores.gov.pt`.

by doing some stitch embroidery. Then I moved to building boxes with ribbons and flowers for napkins, for decorating a child's room or as sewing boxes. There was even an exhibition with those boxes, which had some demand. In addition, I became interested in working with clay. I attended a course and with the kiln my husband offered me, I made Nativity scenes, decorative plates and chimes. However, the opportunity to attend a course on wheat straw embroidery on tulle in the Capelo Crafts School arose. Whenever I had some free time, I carried on perfecting the technique at home. Later, I was encouraged by the Regional Centre for Handicrafts to request my certification."

We shall now analyze the symmetries found in some wheat straw on tulle works created by Isaura Rodrigues (Fig. 10), whose promptitude and responsiveness we appreciate immensely.

Let's begin with the scarf of images (a) and (b). It's easy to identify a half-turn (a 180-degree rotational symmetry). This means that if we turn the scarf upside down its configuration is not altered. This kind of symmetry is very usual not just in arts & crafts but in sidewalks and balconies as well. This abundance has a very practical reason. For example, a centerpiece with a half-turn symmetry has exactly the same configuration if seen from one or the other side of the table, in front of us. The same goes for a sidewalk with this kind of symmetry. As for the scarf, the person wearing it has just to worry about which is the right side of the piece; otherwise, it can be rotated 180 degrees as much as you like for the configuration will not change.

This scarf is an example of a rosette. Due to not presenting reflection symmetries, the scarf of (a) and (b) has C_2 as its symmetry group.

Let's consider some rosettes with mirror symmetries. The skirt (c) has two perpendicular axes of symmetry: a horizontal axis and a vertical axis. In (d), we can take a look to the rosette in more detail. If we place a mirror perpendicularly to the skirt, so that the edge of the mirror is set on the horizontal line (or the vertical line) passing through the center, then we will see that each side of the skirt is indeed the reflection of the other. This means that if we fold the skirt along one of the axes of symmetry, the two halves should overlap completely. It is also easy to see that the figure has a half-turn symmetry, thus D_2 is the symmetry group of this rosette.

Let's take a look to another skirt (e). The centre of the rosette is again of type D_2. However, if we consider the whole visible pattern in the image, it is possible to identify only one vertical axis of symmetry. In addition, if we turn the skirt upside down its configuration is changed (due to the branches of the left and the right). Therefore, the whole visible pattern of the skirt (e) is of type D_1. The same conclusion applies to the example (f).

The skirt of the image (g) is decorated with numerous flowers of 6 petals. Each of these flowers has D_6 as its symmetry group. In fact, it is possible to count 6 repetitions (6 petals), for which the rotation angle should have an amplitude of $360/6 = 60$ degrees (or any of its multiples) as to obtain a symmetry of that flower. We can also identify 6 axes of symmetry (all axes pass through the rota-

Figure 10: Works of Isaura Rodrigues.

tion center; 3 of them cut in half opposite petals and the remaining 3 separate consecutive petals).

Finally, in (h), we identify two motifs that are alternately repeated along a strip. The spacing between consecutive copies of these motifs is always the same. Therefore, this pattern is a frieze, which is characterized by the presence of translational symmetries in one direction. The configuration of the frieze is slightly different if we turn the skirt upside down, so we don't have half-turns. On the other hand, it is possible to identify vertical reflections, whereby F_1^2 is the group of symmetry of this frieze.

5 Angra do Heroísmo: the city of the seven friezes

The sidewalks and squares paved in Portuguese Pavement or Portuguese mosaics are one of the most characteristic aspects of the heritage of many Portuguese towns. We step on them every day but most of the time we do not pay attention to its historical, artistic and geometrical heritage.

Mosaic is as old as the most remote civilization in History. Nevertheless, it was the Roman civilization that took mosaic to the paving of the domus and the villae. In Portugal, its use as a decoration is a nineteenth-century derivation of the Roman way. It was in Lisbon that for the first time, in 1848, paving was used in urban spaces: the Mar Largo project, a composition shaped as waves, was built at the D. Pedro IV Square (Rossio nowadays). But six years before this project, by initiative of Lieutenant-General Eusébio Cândido Cordeiro Pinho Furtado, the narrow streets leading to the São Jorge Castle were paved with white (limestone) and dark (basalt) stones.

Promptly, Portuguese pavement spread to the rest of the continental country and later on to the archipelagos of the Azores and Madeira. It also went beyond the national borders for Portuguese pavior masters were asked to implement and teach their art abroad. This application can be seen in projects such as the S. Sebastião Square in Manaus (Brazil); the famous Calçadão of the Copacabana Beach in Rio; and in Macau, Cape Town (South Africa) and many other places.

In the Azores, artistic paving dates from mid-twentieth-century and replaced the basaltic paving of the sidewalks of the former and main streets of the towns. It also spread to squares and nowadays to private atriums and gardens, bearing different artistic patterns where the black basalt contrasts with the white limestone. For reasons of economy, the prevalence goes for the local black basalt stone and the white limestone (imported from Lisbon) is used on a smaller scale.

We should point out a major achievement for the valuation of the sidewalk as an Azorean heritage, including as a tourism attraction. In June 2014, Angra do Heroísmo (Terceira) reached the status of "City of the Seven Friezes" for having the seven possible types of friezes in its sidewalks, thus following in the footsteps of Lisbon. It is the first Azorean city to achieve this feat and, most likely, the second of Portugal, after Lisbon.

In Fig. 11, it is shown an example of each of the seven types of friezes. One should bear in mind that all friezes have a common property: the translational symmetries in one direction, which implies precisely the repetition of a motif along a strip. For example, in (a) it is possible to identify a basalt parallelogram repeated successively along the strip, with their consecutive copies showing equal spacing between themselves.

The first examples can be found on the sidewalks of Rua de São João (a), Rua da Conceição (b) and Avenida Tenente Coronel José Agostinho (c). Besides the

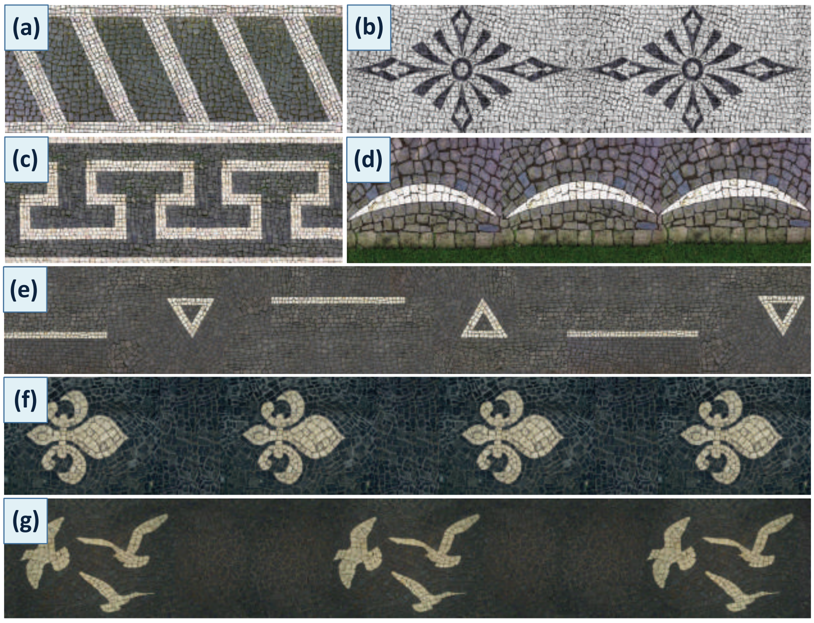

Figure 11: Seven types of friezes.

translational symmetries, these three types of friezes share another property: the half-turn symmetries. If the reader imagines each one of these upside down, the result is that the configuration does not change. The practical effect of this property explains why these three types of friezes can be found so abundantly in our sidewalks, our balconies, and several crafts: if you look to the frieze from one or the other side of the sidewalk its configuration is not altered. Then, what tells them apart? Besides the translational and half-turn symmetries, the first (a) does not possess other symmetries – F_2; the second (b) has horizontal and vertical reflection symmetries – F_2^1; finally, the third (c) shows vertical reflection symmetries and also glide reflection symmetries, which produce a kind of "zigzag" effect similar to our footprints when walking on the sand barefoot – F_2^2.

Let's look at a very peculiar example. Resembling the frieze at Avenida Tenente Coronel José Agostinho (c), the frieze at Rua da Queimada (e) also has glide reflection symmetries. Yet, the latter does not have half-turn symmetries – F_1^3. Should the reader focus on the position of the triangles and the line segments of the frieze at Rua da Queimada (e) and imagine it upside down, you will find out that the configuration you get is different from the original one. So, a new frieze is obtained, with a different disposition of the triangles and the line segments, but keeping the glide reflection symmetries. Interestingly, this new frieze can be spotted farther ahead, at Rua Madre de Deus. Thus, these are two different friezes with the same group of symmetry, since both have only translational and glide reflection symmetries. These are the only examples of this type of frieze in Angra do Heroísmo.

The next type of frieze is characterized by having only translational and vertical reflection symmetries – F_1^2. We can find examples of this type at the Duque da Terceira Garden (d), Rua de Cima de Santa Luzia and a short section of Rua Direita.

Before June 2014, there were only these five types of friezes in Angra do Heroísmo. Two types of friezes were still missing so the city of Angra, a World Heritage Site by UNESCO, could reach the status of the "City of the Seven Friezes". The quest ended precisely in June 2014 when the two friezes missing were inserted in the sidewalk near the Colégio Square: a frieze (f) with translational and horizontal reflection symmetries – F_1^1, and a frieze (g) with just translational symmetries – F_1. The motifs used for implementing these friezes are a creation of Architect Maria João Miranda, aided by Paulo Mendonça. The professional approach of the staff of the municipality of Angra do Heroísmo, lead by Professor Álamo Meneses, and the enthusiasm with which my proposal for achieving this status was welcomed cannot be forgotten. In a nutshell, "Angra, the City of the Seven Friezes" is an achievement that:

- brings added value to the cultural heritage of sidewalk paving;
- can be a tool for teachers for field trips (since the theme of symmetries is part of the present syllabus), connecting Mathematics to every day life;
- brings forth the opportunity to discover a growing tourism segment, the Mathematical Tourism ("Mathourism");
- is a fine example of the potential of Recreational Mathematics.

6 The endless search for symmetry

The Holy Spirit festivities take place on all the islands of the Azores and beyond that, in the diaspora. The Holy Spirit cult is one of the oldest and best-known practices of popular Catholic religiosity. The cult originated on mainland Portugal (probably with queen Santa Isabel) and were brought to the Azores by the early settlers. Gastronomically, the hosts of the festivities offer the traditional soup, the roast beef, *massa sovada* (a sweet bread) and *arroz doce* (a rice pudding). This rice pudding is usually decorated with cinnamon. And why not take the chance to replicate here the seven types of friezes? In Fig. 12, we show some examples of rice pudding's decoration that took place in a Holy Spirit festivity, in June 2014, for which there was no limit to mathematical imagination!

We wish to thank to Edna Soares and Pedro Soares, who hosted the Holy Spirit festivity, and also Goreti Rosa Carvalho, who cooked the Rice Pudding.

The half-turn friezes were selected as to pay homage to the three Azorean cities with more frieze types. There is the sidewalk of Rua Dr. Aristides da Mota, in Ponta Delgada (f); the sidewalk at Praça da República, in Horta (h); and a section of the sidewalk of Rua da Sé, in Angra do Heroísmo (j).

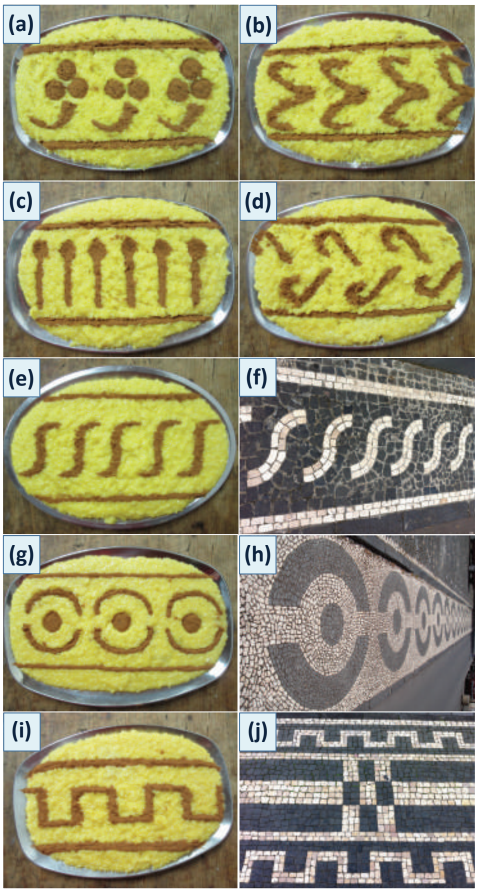

Figure 12: Symmetry groups: (a) – F_1; (b) – F_1^1; (c) – F_1^2; (d) – F_1^3; (e) and (f) – F_2; (g) and (h) – F_2^1; (i) and (j) – F_2^2.

Furthermore, the types of friezes selected are very usual in the aforementioned cities:

- In Ponta Delgada the friezes F_2 are very common: at Rua Dr. Guilherme Poças, at Av. Gaspar Frutuoso, at Largo da Matriz, at Portas da Cidade, at Rua 6 de Junho, at Largo de Camões and at Rua dos Clérigos;

- In Horta there are many friezes with symmetry group F_2^1: at Alameda Barão de Roches, at Largo Duque d'Ávila e Bolama, at Rua Walter Bensaúde, at Rua Conselheiro Medeiros and at Rua José Azevedo;

- In Angra do Heroísmo one can find many friezes F_2^2: at Rua Beato João Batista Machado, at Canada dos Melancólicos, at Rua Dr. Henrique Braz, at Rua Dr. Luís Ribeiro, at Rua Padre Manuel Joaquim Máximo and at Avenida Tenente Coronel José Agostinho.

We end with a brief visual catalog comprising a few more examples of the symmetries that can be found in the Azorean sidewalks.

Some of these examples shall be present in a deck of cards bearing the sidewalks of the Azores to be launched soon by the Ludus Association, with co-authorship of Alda Carvalho, Carlos Pereira dos Santos, Jorge Nuno Silva and Ricardo Cunha Teixeira.

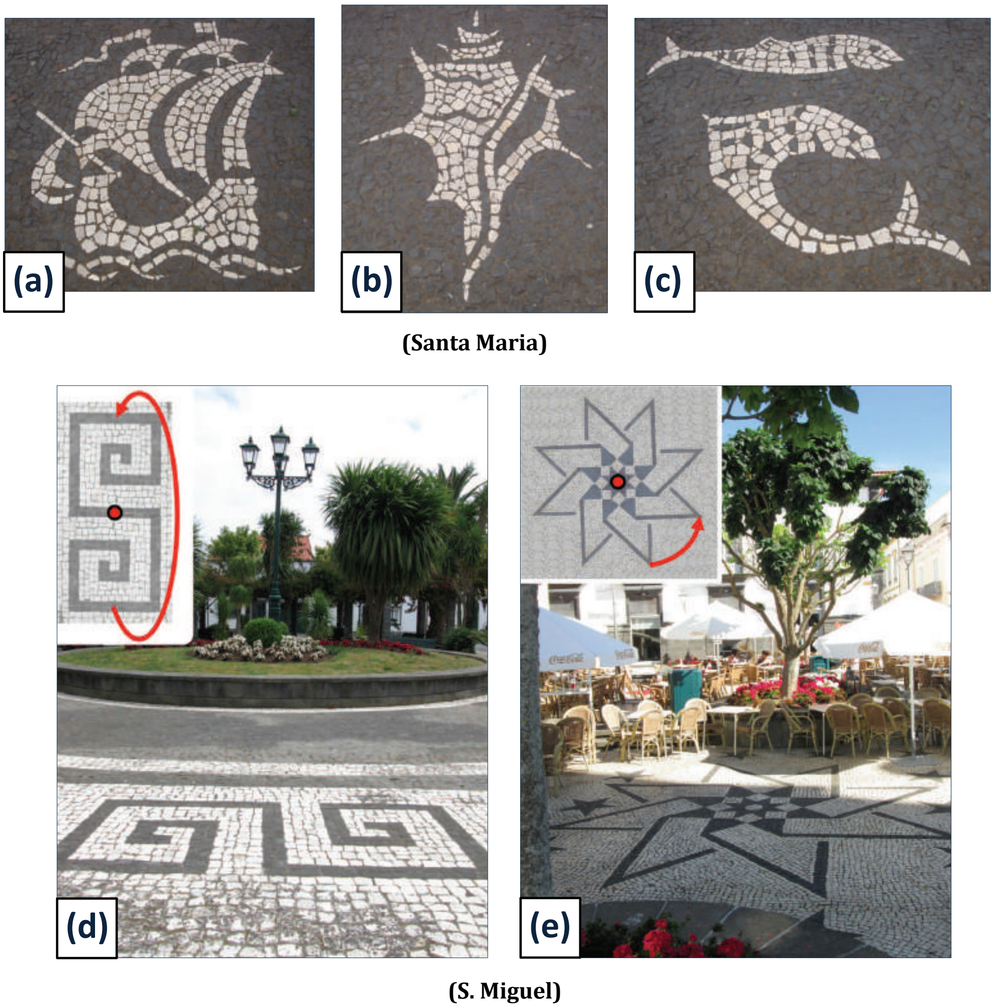

Figure 13: Symmetry groups: (a), (b) and (c) – C_1; (d) – C_2; (e) – C_8.

Figure 14: Symmetry groups: (a), (b) and (c) – D_1; (d) – D_6; (e) – D_8.

(Terceira)

(Graciosa)

(S. Miguel)

(Faial)

Figure 15: Symmetry groups: (a) – F_1; (b) – F_1^1; (c) – F_1^2; (d) – F_1^3.

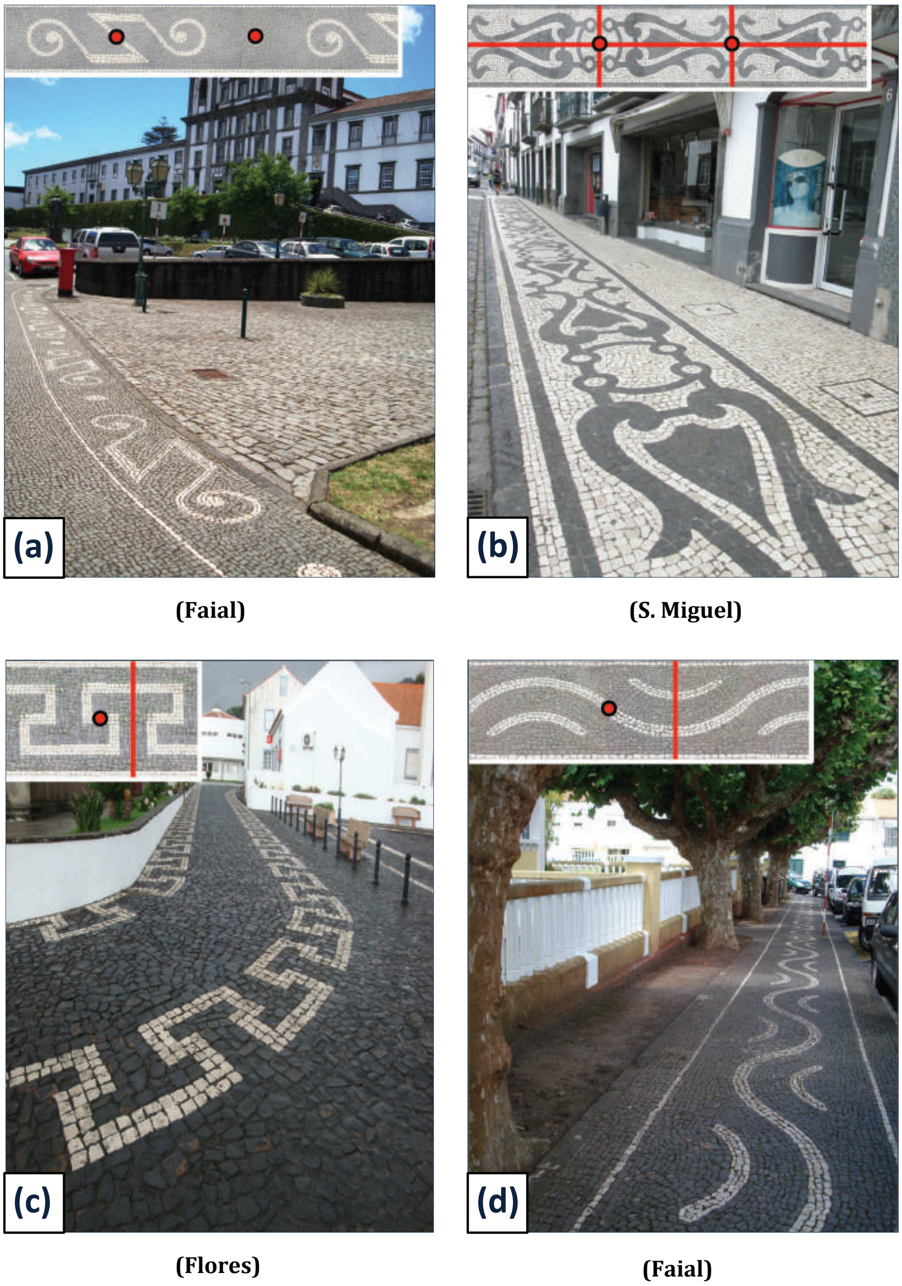

Figure 16: Symmetry groups: (a) – F_2; (b) – F_2^1; (c) – F_2^2; (d) – F_2^2.

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