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SOME EARLY TOPOLOGICAL PUZZLES
PART 1
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Ludus



CIUHCT

Games and Puzzles

SOME EARLY TOPOLOGICAL PUZZLES

PART 1

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An earlier version of this was presented at the Fourth *Gathering for Gardner*, Atlanta, 19 Feb 2000. Because there were too many pictures, it was not included in the proceedings of that meeting [16]. I have scanned in the 24 pictures that I showed as OHP slides and seven extra pictures and corresponding material. I have now inserted these in this version of this work, with the picture name after each Figure reference.

The most important development in the field is that Dario Uri has systematically examined Part Two of the Pacioli MS which I had found difficult [17]. He has discovered that the following puzzles occur. The Alliance or Victoria Puzzle, Cap. C, CII & CIII. Solomon's Seal, Cap. CI. The Cherries Puzzle, Cap. CIII & CX (the latter being with a tube). The Chinese Wallet, Cap. CXXXII. As mentioned below, I had already noticed Cap. C CI, and a bit later I noticed CIII, but had set the transcription aside to work on later. In addition, Dario has found that Cap. CVII is the Chinese Rings! Further, Cap. CXVIII is the problem of joining three castles to three wells by paths that do not cross; Cap. CXVII is a trick of removing a ring from a loop between a person's thumbs – cf the top of Figure 12: Schwenter410; Cap. CXXI is the trick where strings are just looped inside a ball so they can be pulled away; Cap. CXXIX is the problem of making a support from three knives. As far as we know, these are the earliest appearance of all these puzzles. There are about 140 problems in this Part! Unfortunately, though Pacioli refers to diagrams, the only diagram in the MS is for Cap. C. Perhaps Dario will find the missing drawings in the library at Bologna. Dario has found that Cap. CXXIX refers to Pacioli being shown the problem on 1 Apr 1509, so this MS should be dated as 1510?

I have just added much further material on two topological patterns: The Star of David and The Borromean Rings; The Möbius Strip; which go back rather further than I knew.

Abstract: *For some time, I considered the 1723-1725 edition of Ozanam's Récréations mathématiques et physiques as the first book to cover topological puzzles in detail and I only knew of a few earlier examples. Ozanam certainly gives many more examples than any previous book. In the last few years, I have discovered some early sources which show several topological puzzles as being considerably older than I previously knew. Here I show them.*

Key-words: African beads puzzle, Chinese wallet, flick-flack, Jacob's ladder, Möbius strip, Solomon's seal, star of David, topological puzzles, Victoria puzzle.

1 The Star of David

The hexagram pattern of two overlapping equilateral triangles, Mogen David, is known from antiquity, both from Hebrew and other cultures, including Turkish, Hindu and Japanese! - see the Wikipedia entry for numerous references back to about 0 CE. It was sometimes called Solomon's Shield or Seal. Early examples do not show the triangles interlocking, as in this example from the Leningrad Codex of 1008 (Figure 1).



Figure 1: From the Leningrad Codex of 1008.

Interlocking does go back to the 12C - as seen here in an 1141 mosaic at Santi Maria e Donato, Murano, Venice ([11] p.49) (Figure 2) and a 1641 mosaic at San Salvatore, Venice (ibid., p. 85) (Figure 3).



Figure 2: An 1141 mosaic at Santi Maria e Donato, Murano, Venice.



Figure 3: 1641 mosaic at San Salvatore, Venice.

At various points, this was extended in several different directions.

First: With interlocking squares. c4C mosaic from Lenhay Green, Sherborne, Dorset ([7], plate 44, p. 55) (Figure 4).



Figure 4: c4C mosaic from Lenhay Green, Sherborne, Dorset.

Second: Into a knot pattern of two interlocked loops, also known as “Solomon’s Knot” or “Seal”. Floor mosaic from Fishbourne, c200 ([10], p. 26) (Figure 5).



Figure 5: Floor mosaic from Fishbourne, c200.

And [11], p. 51, from an 1141 mosaic at Santi Maria e Donato (Figure 6).



Figure 6: From an 1141 mosaic at Santi Maria e Donato.

Second (A): This is often further embellished by a third interlaced loop, e.g. from c315 Christian mosaics at Aquileia ([8], pl. 41) (Figure 7).



Figure 7: c315 Christian mosaics at Aquileia.

Third: To have three interlocking triangles, in the form known as the “Borromean Rings”.



Figure 8: Photo of my example (Borromean Rings).

In the Figure 8 each loop is made from six right-angle Tangle pieces which make models of the cyclohexane molecule isomers - these are in the “chair” configuration. In this pattern of three rings, no two are actually linked, but all three are. It is part of the coat of arms of the Borromeo family, who are counts of the area north of Milan since the 15C. However a considerably earlier version occurs as “Odin’s Triangle” or “Walknot” in Viking culture.

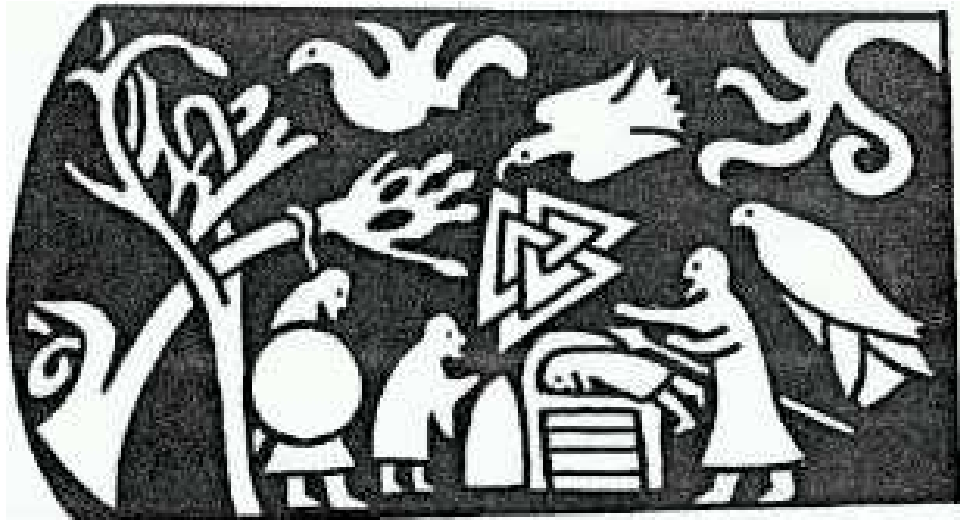
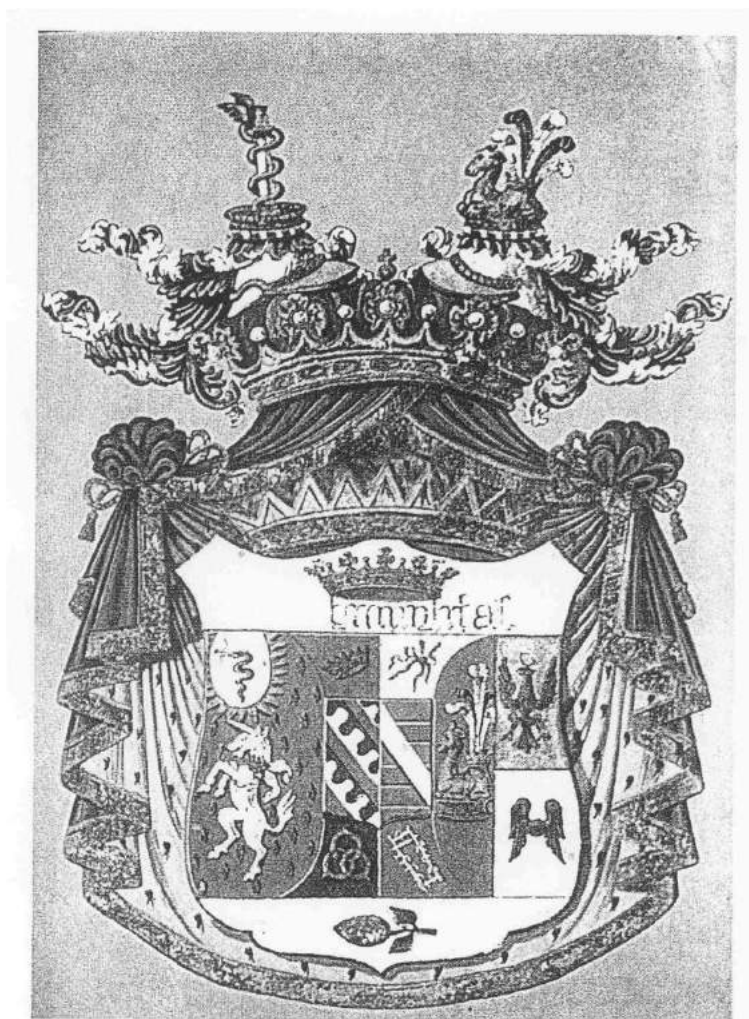


Figure 9: 9C Larbro Picture Stone from the island of Gotland.

The 9C Larbro Picture Stone from the island of Gotland is now in the Historiska Museum, Stockholm (Figure 9). A similar carving is in the Viking Ships Museum, Oslo.



- Stemma della famiglia Borromeo
- Armoiries de la famille Borromeo
- Coat of arms of the family Borromeo
- Wappen der Familie Borromeo
- Wapen van de Familie Borromeo

Figure 10: Borromeo Crest. *Albero Genealogico Storico Biografico della nobile Famiglia Borromeo* (1903). This is available at: www.verbanensia.org. This says it is copied from a manuscript of the archivist Pietro Canetta, with a footnote: *Il Bandello*, p. 243, vol. VIII. I suspect that this refers to a publication of the MS. This simply says that the three rings represent the three houses of Sforza, Visconti and Borromeo which are joined by marriages. (Thanks to Dario Uri [email of 17 Jul 2001] for this source).

The Golfo Borromeo and the Borromeo Islands are in Lago Maggiore, off the town of Stresa, NW of Milan. In the 16 and 17C, the Counts of Borromeo built a baroque palace and gardens on the main island, Isola Bella (or Isola Borromeo). The Borromeo rings can be seen in many places in the palace and gardens, including the sides of the flower pots! Although the Rings have been described as a symbol of the Trinity, I don't know how they came to be part of the Borromeo crest, though the guide book describes some of the other features of the crest. (Thanks to Alan and Philippa Collins for the information and loan of the guide book.) Perhaps the most famous member of the family was San Carlo Borromeo (1538-1584), Archbishop of Milan and a leader of the Counter-Reformation, but he does not seem to have used the rings in his crest.

Hornung, plates 10, 20, 24 & 39 are examples of Borromeo rings, shown in the Figure 11. These designs have no descriptions and the only dating is in the Publisher's Note which says such designs were common in the 12C-17C.

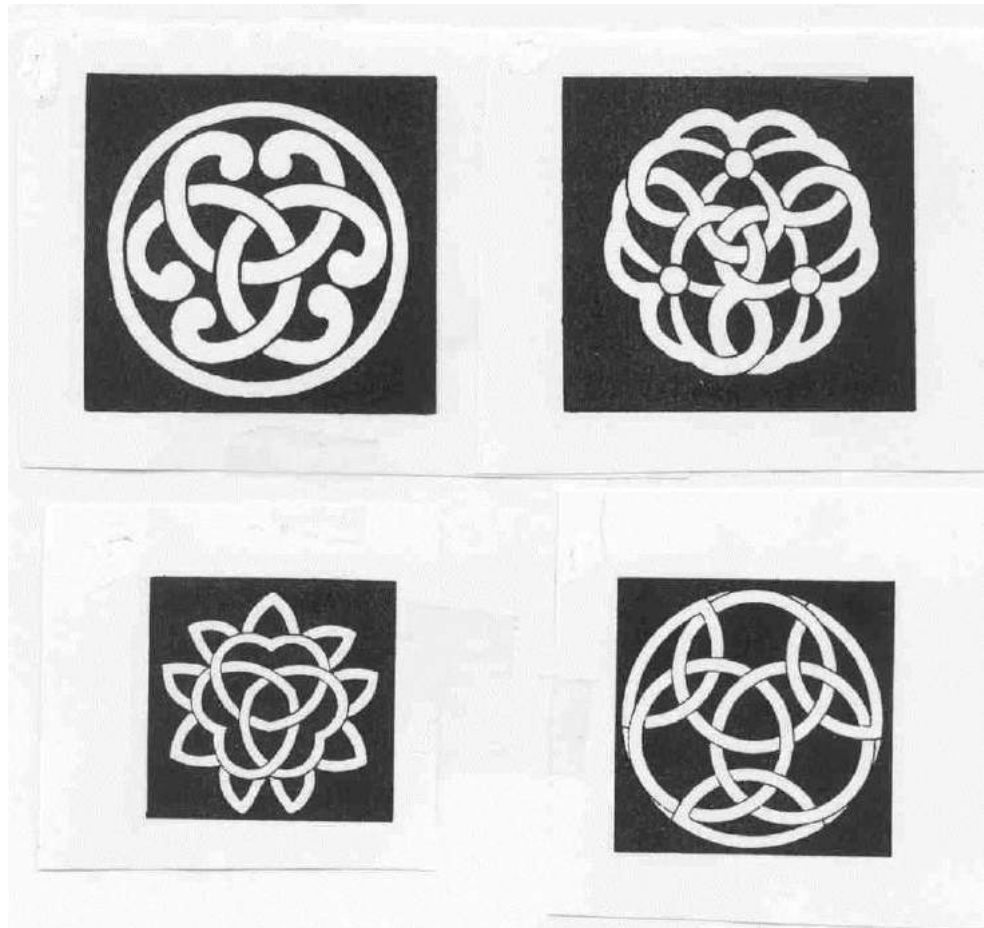


Figure 11: Japanese patterns from Hornung.

The pattern continues to intrigue - see the Wikipedia entry for more examples. The 20C British mathematical sculptor John Robinson has made several versions with interlocked triangles, squares and rhombi. This example, called “Intuition”, is outside the Isaac Newton Institute in Cambridge, England (Figure 12).



Figure 12: Intuition.

In 2006, the pattern has been adopted as the logo of the International Mathematical Union (Figure 13).



Figure 13: IMU Logo.

2 The Möbius Strip

This is well known, but I was surprised to recently discover its age (Figures 14 and 15).



Figure 14: Mosaic of Orpheus from c3C in the Museum of Pagan Art, Arles.



Figure 15: Early 3C Mosaic from Sentium (now Sassoferrato, Umbria).

This is now in the Glyptothek (Figure 15), Munich, discovered by Charles Seife in 2002.

There are a number of practical uses for the Möbius strip, but the most unusual is as a non-inductive electrical resistor, patented by Richard L. Davis in 1966 - US Patent 32674906 A (Figure 16).

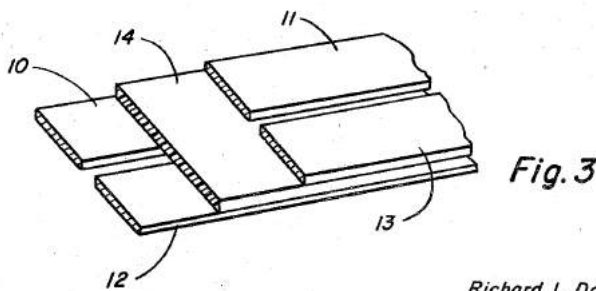
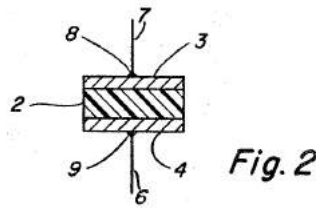
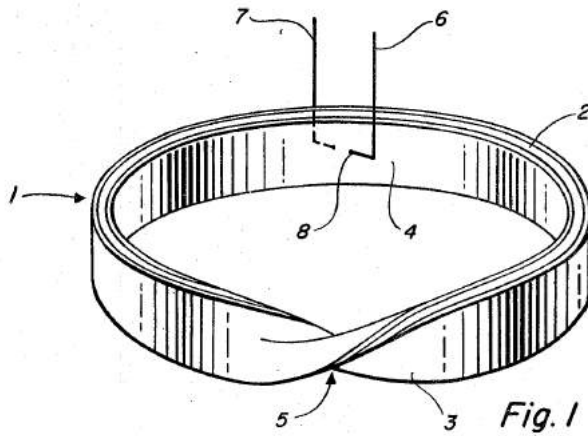
Aug. 16, 1966

R. L. DAVIS

3,267,406

NON-INDUCTIVE ELECTRICAL RESISTOR

Filed May 1, 1964



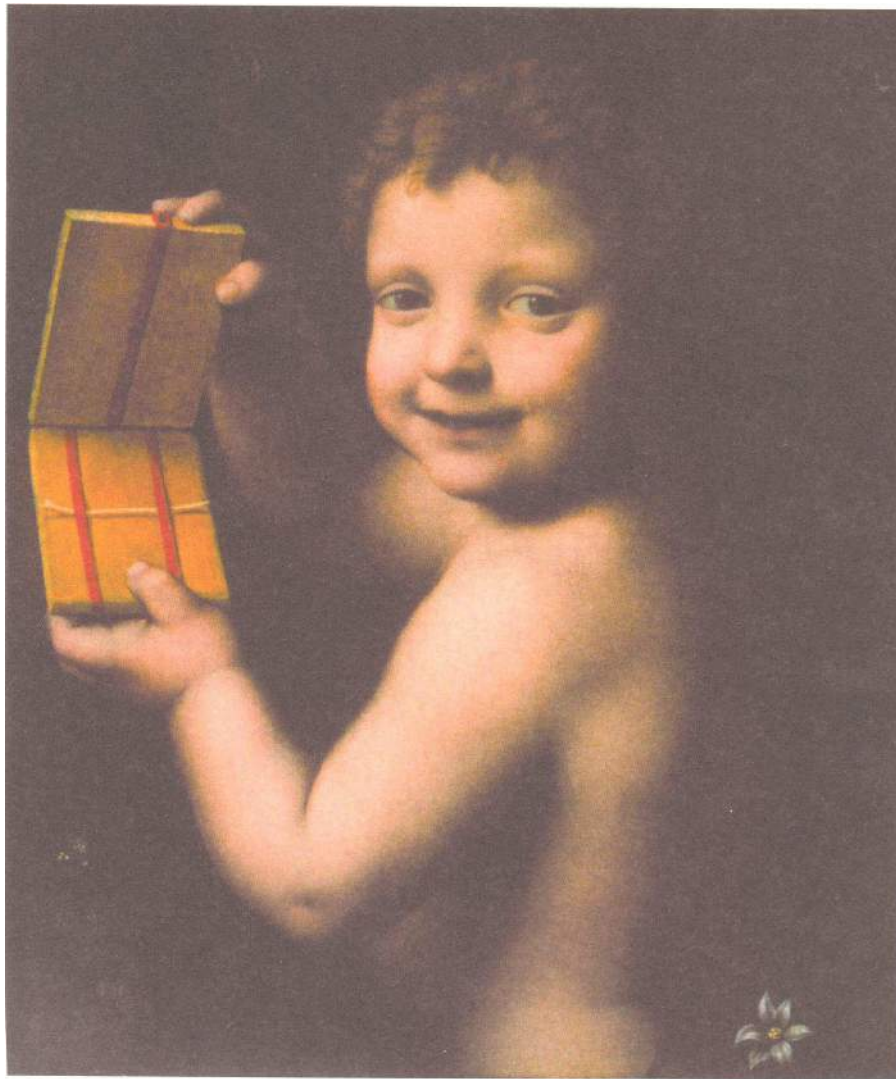
Richard L. Davis
INVENTOR.

BY
Richard A. Anderson
ATTORNEY

Figure 16: Davis' patent.

3 The Chinese Wallet or Flick-Flack or Jacob's Ladder

In May 1998, Peter Hajek reported seeing a painting at Hampton Court showing this (Figure 17). Some research in art history books turned up a picture of it and a reference to another depiction, probably earlier.



Bernardino Luini's Boy with a Puzzle, ca. 1520, oil on wood, 15" × 13", Peterborough, Elton Hall, the Proby Collection

Figure 17: Luini Colour.

This is a painting (Figure 17) by the fairly well known Lombard follower of Leonardo, Bernardino Luini, born c1470 and last known in 1533. It is in the Proby Collection at Elton Hall, a stately home at Peterborough, Cambridgeshire. It is 15" by 13" (38 cm by 33 cm). The picture is variously cited as "A Boy with a Toy" or "Cherub with a Game of Patience". There is no indication of its date, but the middle of Luini's working life is c1520. A description says the tapes holding the boards together are apparently holding a straw, but doesn't seem to recognize the object. The painting was exhibited in London in 1898 and a contemporary review in a German journal calls it a *Taschenspielerstückchen*, a little juggler's trick – but recall that juggler was long a synonym for magician – with two boards which allow one to vanish the straw. This version appeared in an article by Volker Huber [5].

In 2005, I saw: Tancred Borenius & J. V. Hodgson; A Catalogue of the Pictures at Elton Hall in Huntingdonshire in the possession of Colonel Douglas James Proby; The Medici Society, London, 1924. Plate 5 is a B&W photo: Boy with a Puzzle. The facing p. 11 describes the picture. This says Luini died in 1532 and the picture is 17" by 13" (43 x 35 cm). It is stated to have been in the Arundel Collection, but there is no evidence for this. It was owned by Sir William Hamilton and William Beckford. It is mentioned by Bernard Berenson ([2], p. 251) and is the frontispiece of Luca Beltrami ([1], pp. 563-564). The same child appears in another Luini painting, Christ and the Baptist as Children. The authors identify the "straw" as a fishbone and say it will disappear.

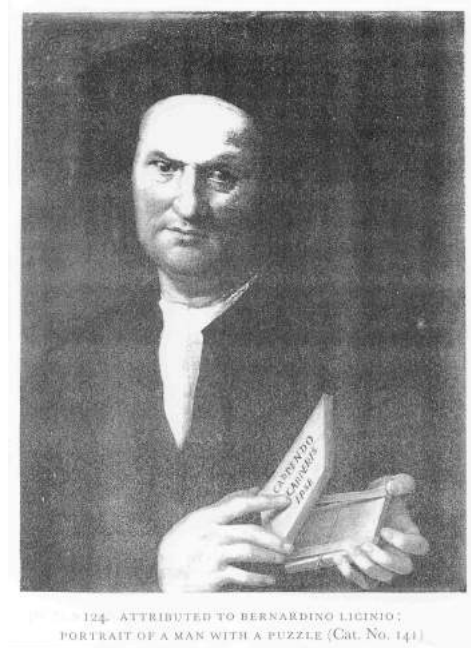


Figure 18: Licinio Painting.

This is the picture (Figure 18) that Peter Hajek saw in the Picture Gallery at

Hampton Court Palace - but I saw it at Windsor Castle in about 2012. It is attributed to Bernardino Licinio, a Venetian painter born about 1485 and last known in 1549. It is 22" by 18" (56 cm by 45 cm). It is described and illustrated in John Shearman, [13], where it is called "Portrait of a Man with a Puzzle". It is very similar to another painting known to be by Licinio and dated 1524, so this is probably c1524 and hence a bit later than the Luini. Shearman cites the Luini painting and Pauli's notice of it. The description says the binding tapes are red, as in the Luini, and both show something like a straw being trapped in the wallet, which suggest some connection between the two pictures, though it may just be that this toy was then being produced in or imported to North Italy and was customarily made with red tapes. On the toy is an inscription: *Carpendo Carperis Ipse* (roughly: Snapping snaps the snapper), but Shearman says it definitely appears to be an addition, though its paint is not noticeably newer than the rest of the painting. Shearman says the toy comprises "three or more rectangles. . .", though both paintings clearly show just two pieces.

*Very Cunning Tablets Adorned with
Strings of Different Colors, Under or
Outside of Which You May, if You
Wish, Make a Wand be Seen*

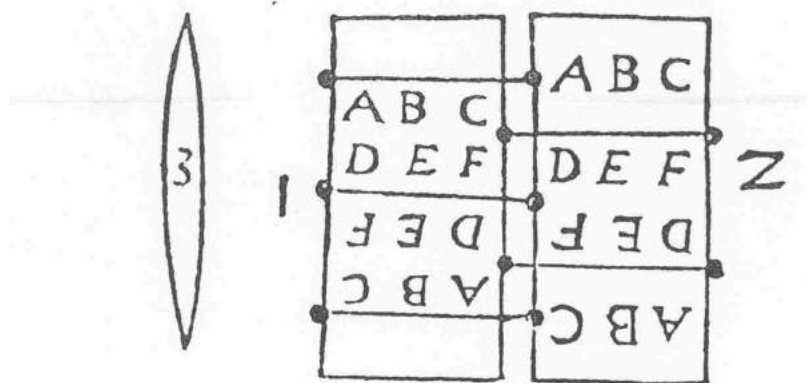


Figure 19: Prevost 139.

A little later in the 16C, we have the first known published version of the "Chinese Wallet" ([9], p. 139). This is the first book devoted to magic and conjuring. It is rather rare - only five copies are known - and a 1987 facsimile is out of

print. Fortunately, it has been recently translated into English as *Clever and Pleasant Inventions - Part One - Containing Numerous Games of Recreations and Feats of Agility, by Which One May Discover the Trickery of Jugglers and Charlatans*, and published in 1998. My thanks to Bill Kalush for bringing this to my attention. The figure is taken from the new edition (Figure 19).

Another recently received source is Schwenter's book of 1636, which has a version ([12], part 15, exercise 27, pp. 551-552) (Figure 20). I read this as *Ein Einmaul*, but Huber's article, above, reads it as *Ein Ginmaul* [yawning mouth].



Figure 20: Schwenter 551.

And another early source is Witgeest (Figure 21). However, most of the interesting material is not in the first edition of 1679 of which there is a recent facsimile. The new material apparently occurs in the 2nd ed. of 1682 and this is so extensively revised and retitled as to constitute a new book. I only have some photocopies from the 3rd ed. of 1686, sent by Jerry Slocum. There were many further editions, both in Dutch and in German. Much of the new material seems to be derived from Schwenter, but here it is clearly different. ([15], prob. 66, pp. 49-50).

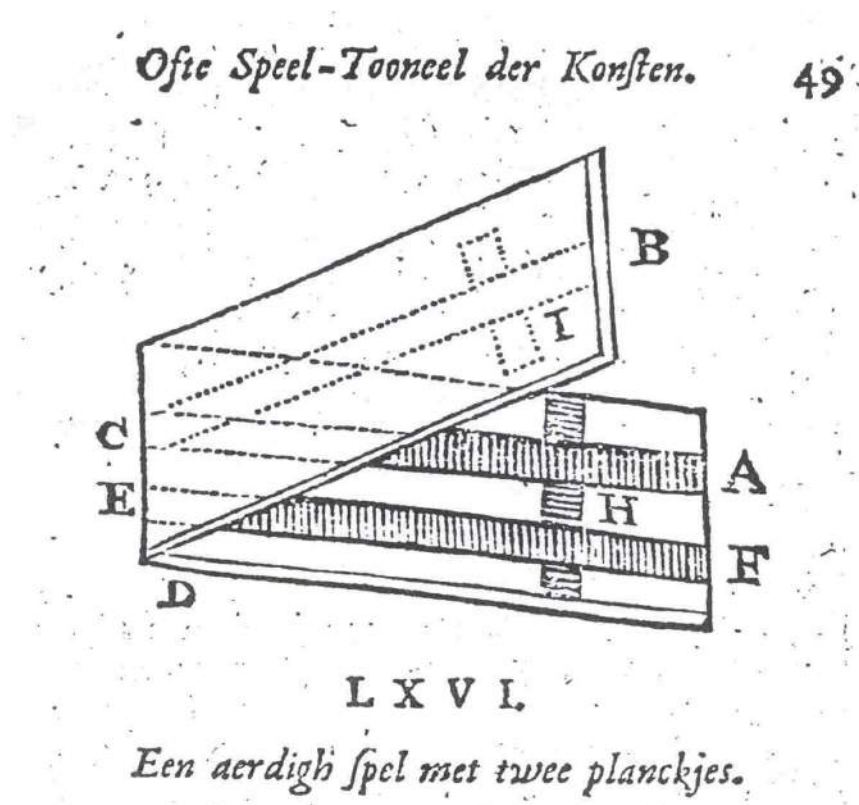
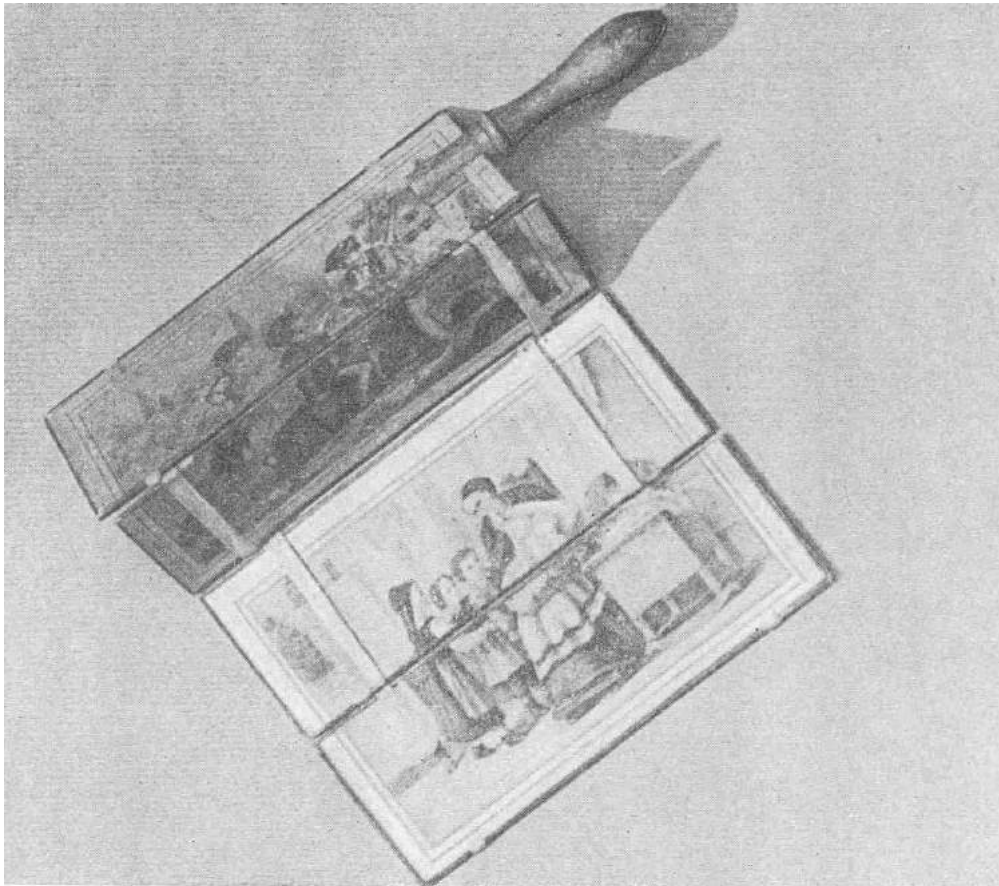


Figure 21: Witgeest 49.

I don't know when the toy advanced to having more boards and becoming the "Jacob's Ladder". Ozanam does not give any version of the toy. I distinctly recall having seen a painting of a gentlemen holding one with more parts but my note of it is buried somewhere. Later thought indicates that version had Victoria and Albert on the two sides. The first example I have record of is a c1850 example of a "Hand-operated game of changing pictures" illustrated in ([4], plate 6 on p. 24) (Figure 22).



6. Hand-operated game of changing pictures, c. 1850

Figure 22: Daiken 24.

4 The Alliance or Victoria Puzzle

This does occur in Ozanam [[6], vol. IV, prob. 31, p. 435 & fig. 37, plater 11 (13)] and I show it first as most later versions look much like this (Figure 23).

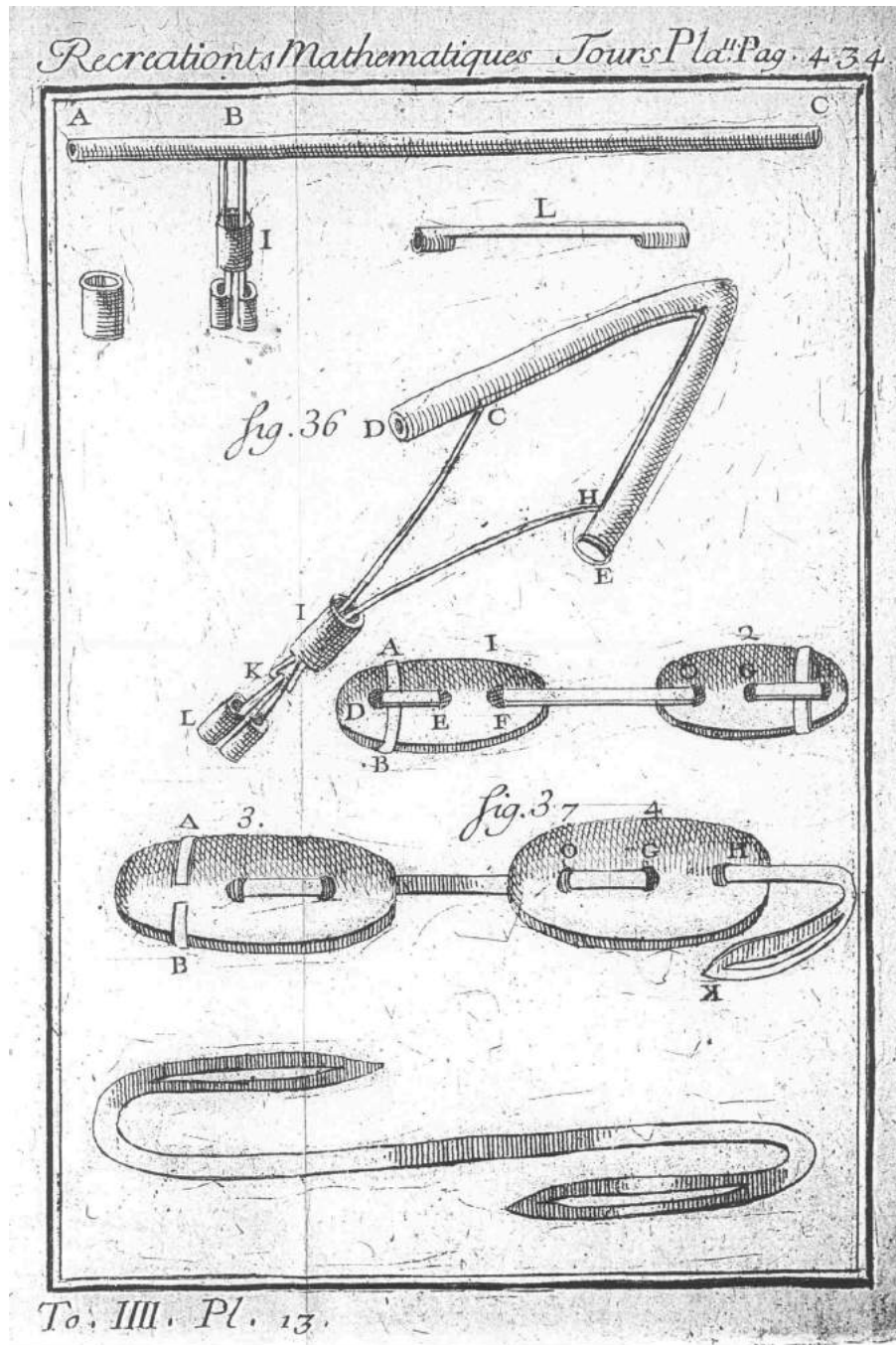


Figure 23: Ozanam Prob 31.

In 1557, Cardan shows both two hole and three hole versions but Cardan's Latin is generally cryptic and I didn't really recognise what the pictures were for until I saw them repeated in the 1660 English version of Wecker [[14], p. 338 (This is in Book XVIII and I don't know if it appeared in the 1582 ed.)] (Figure 24).

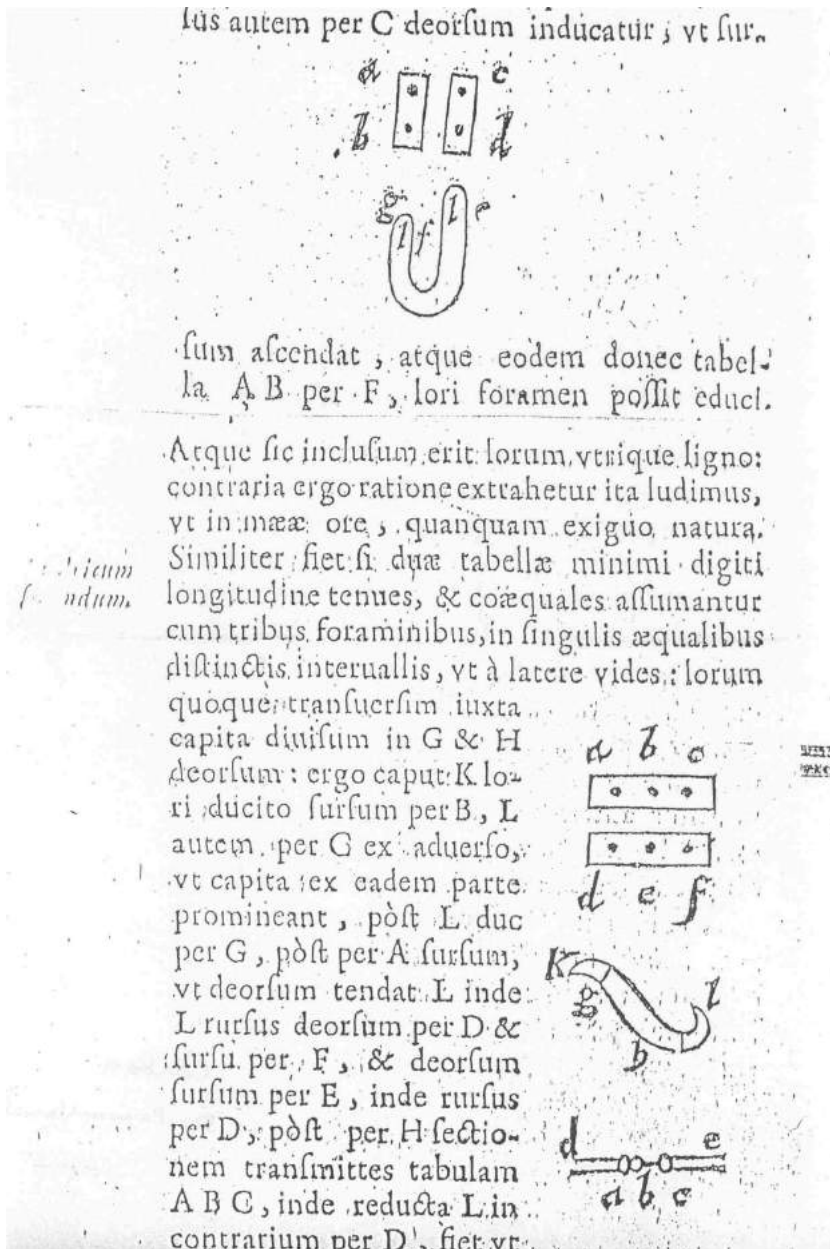


Figure 24: Cardan, [3], vol. III, pp. 245-246.

More recently, I have discovered a marginal drawing in Luca Pacioli's *De Viribus Quantitatis* MS of c1500 [Pacioli, ff. 206r - 206v, Part 2, Capitulo C] (Figure 25) *De cavare una stecca de un filo per 3 fori* (To remove a stick from a string through 3 holes). On f. 206r is the marginal drawing clearly showing the string through three holes in one board.

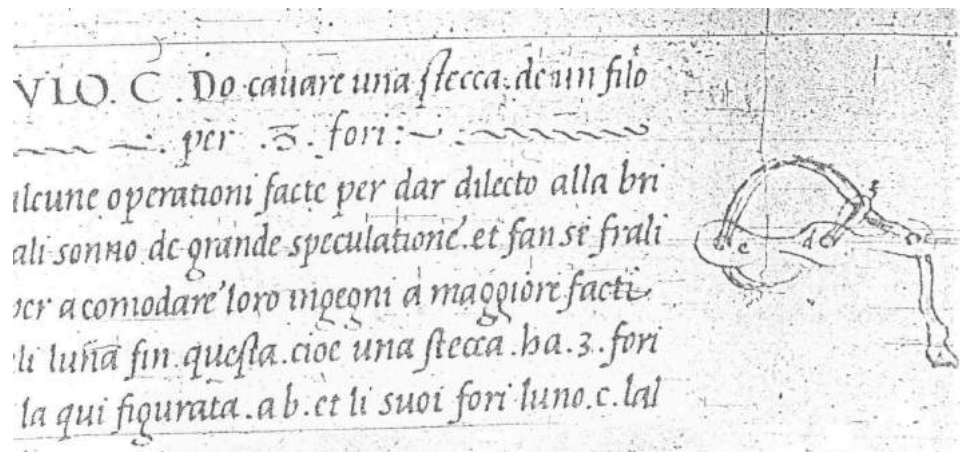


Figure 25: Pacioli 206r.

Both two hole and three hole versions occur in Prévost in 1584 ([9], pp. 133-136) (Figures 26 and 27).

*To Enclose Two Tiny Pieces of Wood
with Two Straps,⁹⁰ So That One May
Not Take Them Out Without Breaking
the Wood or the Straps*

Have constructed two small, straight, and long pieces of wood, as you see drawn hereafter, each one having two round holes near the ends, which are marked with the four letters A, B, C, and D. Then take a slim leather strap of a

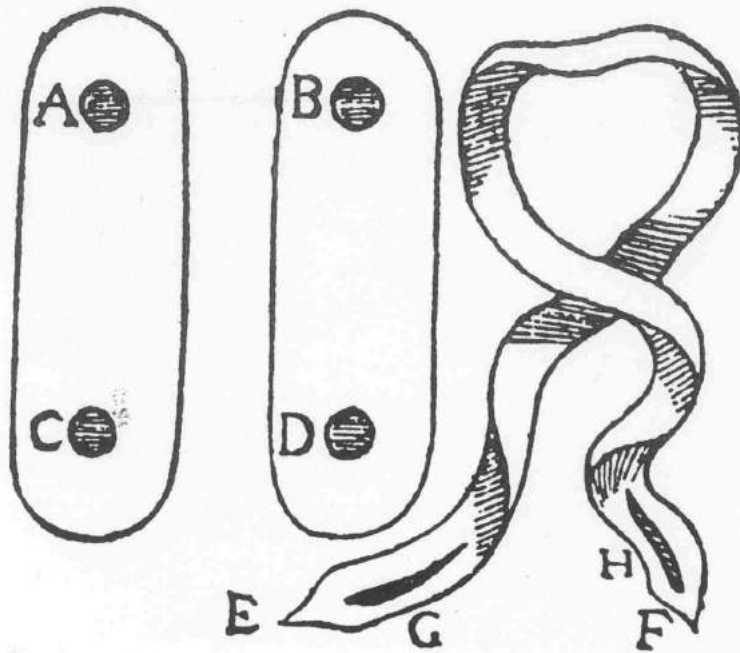
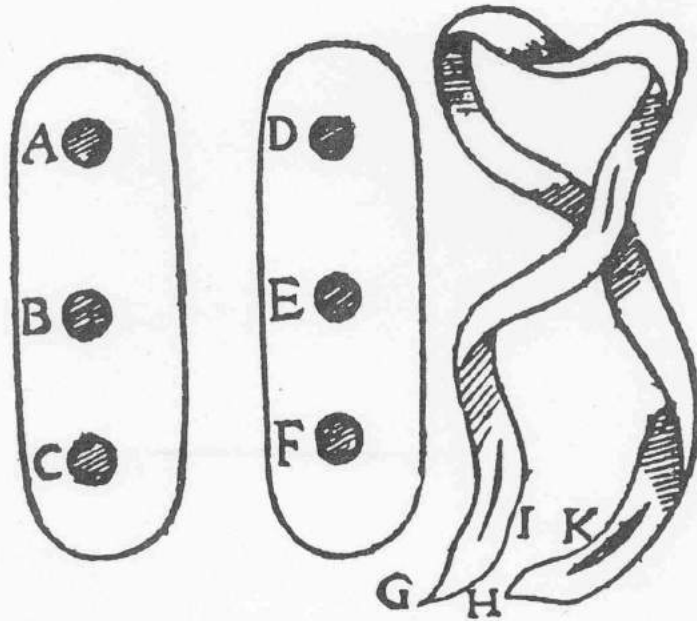


Figure 26: Prévost 133.

G through hole A, and end H through hole B, so that the two ends shall be on one side. Then pass end G through slit K and through hole C.



After this, to enclose the other piece, in a similar way pass end G through D and F, then through E and again through D, where the end is, holding the other piece, which you shall pass through slit I, and you shall pull through the strap from hole D. And the end shall be thoroughly caught in this piece as is the other end

[135]

Figure 27: Prévost 135.

Schwenter gives a two hole version ([12], Part 10, exercise 29, pp. 410-411) (Figure 28).

Fingern bey
hinlegen da
ganz herum
ander thun

Weil solches fallens vrsach ein jeder / so es p
sehen wird / ist die demonstration hieher zu s

Die XXIX Art
Zwey Brättlein mit einer Nestel so ol
also zusammen zu fassen / daß sie nic
zerrissen mögen auffgel.
Schneid zwey Brättlein / vngesähr ein
POLL lang / jedes mit ein viereckichten löchlein
zu sehen: Nimb ein Nestel ohne steffe / schen
darein / ein
oben vnd
stecke das
ziehe es di
das eine ei
der Nests
Das and
Nestel vo
nen bey'de
schramm

hinauß / fahr mit dem Brättlein A B dadurck

Figure 28: Schwenter 410.

([15], prob. 44, p. 35-36) is a two hole version taken from Schwenter (Figure 29).

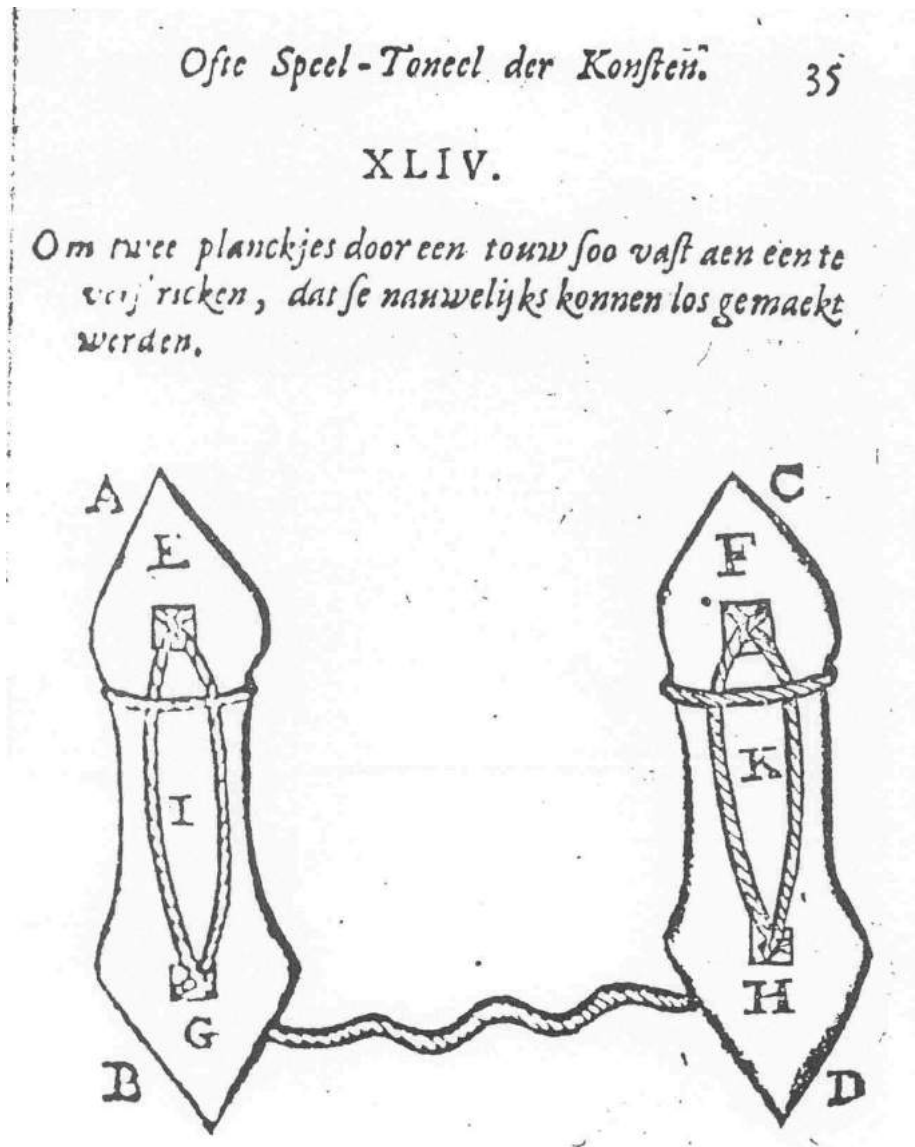


Figure 29: Witgeest 35.

5 Solomon’s Seal or African Beads Puzzle

Again let us start with the version in the Ozanam [6], vol. IV, prob. 40, pp. 439-440 & fig. 47, plate 14 (16) (Figure 30), which calls it *Le Sigillum Salomonis*, or *Sceau de Salomon*. But this doesn’t seem to relate to the shape called “Solomon’s Seal” which we saw earlier.

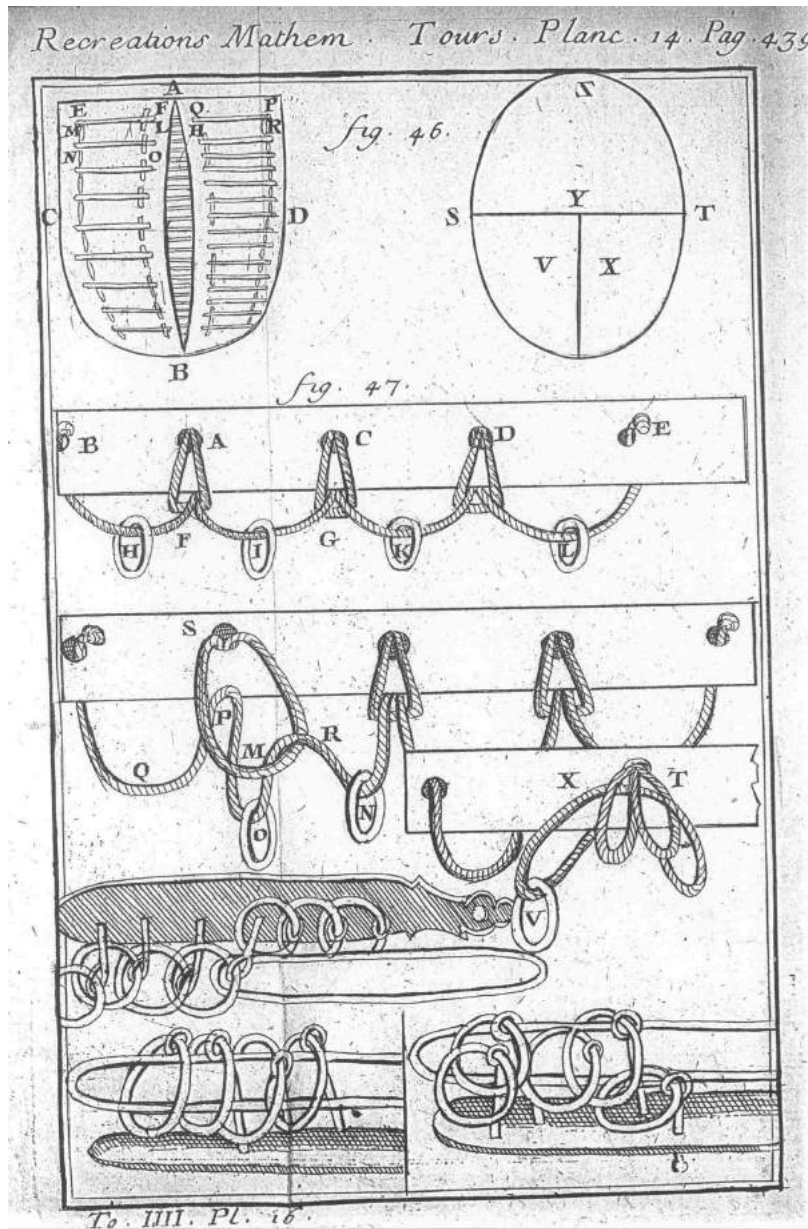


Figure 30: Ozanam Prob 40.

I was pleased to recently find this in Schwenter ([12], Part 10, exercise 27, pp. 408-410).

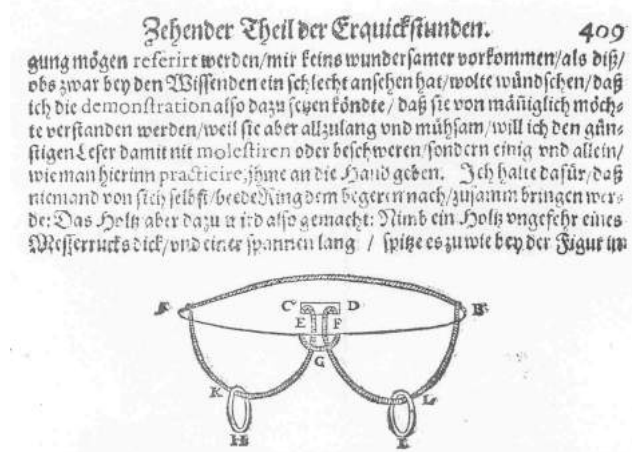


Figure 31: Schwenter 409.

Witgeest ([12], prob. 43, pp. 33-34) (Figure 32) is clearly taken from Schwenter.

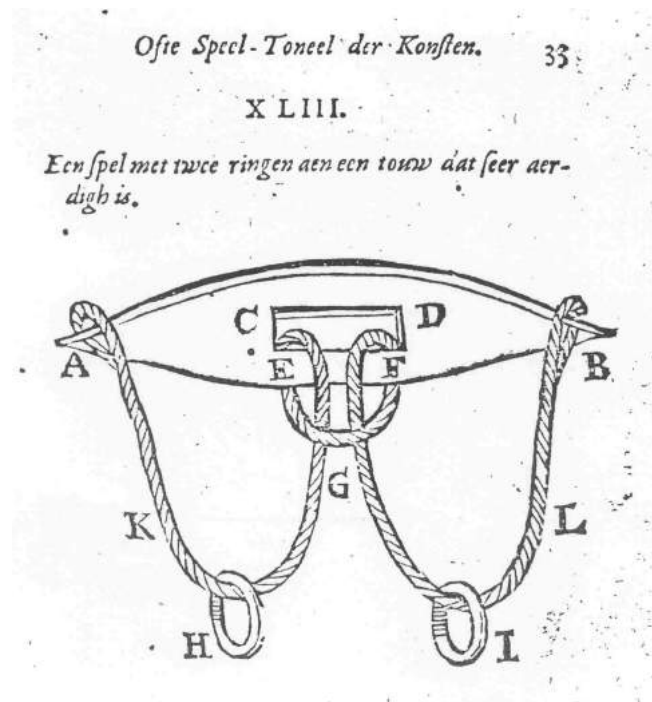


Figure 32: Witgeest 33.

While researching this work, I read some of the text adjacent to the Pacioli cited above. I had been putting off working on this because there is a lot of it and it is not always clear since there are rarely any pictures. Though I haven't yet made a detailed translation, it seems clear that [Pacioli, ff. 206v-207r, Part 2, Capitolo CI]: *De un altro filo pur in 3 fori in la stecca con unambra per sacca far le andare' tutte in una* is describing this same puzzle. The chapter titles vary between the actual problem and the Table of Contents and the latter shows that "unambra" should be "una ambra". Sacca means pocket or bay or inlet and it seems clear he means a loop which has that sort of shape. Ambra is amber, but seems to mean an amber bead here. So the chapter title can be translated as: "On another string also in three holes in the stick with one bead per loop, make all of them go onto one". Sadly there is no picture.

[I later made an inquiry on NOBNET about why this puzzle has recently been called an African puzzle and this turned up the following.

R. P. Lelong. "Casse-tête guerzé". *Notes Africaines* 22 (Apr 1944) 1. Not Yet Seen – cited and described by Béart. Says M. Gienger found the variant with an extra ring encircling both loops in the forest of the Ivory Coast in 1940, named *kpala kpala powa* [body of a toucan] or *kpa kpa powa* [body of a parrot].

Paul Niewenglowski. Bulletin de l'IFAN [Institut Français d'Afrique Noire] 14:1 (Jan 1952). Not Yet Seen – cited and described by Béart. Describes his invention of an interesting, rather simpler, variant as a result of seeing a standard version from Béart.

Charles Béart. *Jeux et jouets de l'ouest africain*. Tome I. Mémoires de l'Institut Français d'Afrique Noire, No. 42. IFAN, Dakar, Senegal, 1955. pp. 413-418 discusses and carefully illustrates several versions. The standard version, but with several beads on one loop, is called *pèn* and is common in the forests of Guinea and Ivory Coast. Describes variants of Gienger/Lelong and Niewenglowski.

Fred Grunfield. *Games of the World*. Ballantine, NY, 1975. On p. 267, he calls this "African String Game", but gives no reference. Probably based on Béart.

Pieter van Delft & Jack Botermans. *Creative Puzzles of the World*. Abrams, New York, 1978. African ball puzzles. "It was once used in magic rites by tribes living in the jungles of the Ivory Coast. The puzzle is still used for amusement in this part of Africa, not only by the people who inhabit the remote outlying areas but also by city dwellers. . . . The puzzles were not restricted to this part of Africa. Variations may be found in Guinea, and some . . . were made in China." No reference given, but I suspect it must come from Béart, although this is not listed in their bibliography. My thanks to Mark Peters for the reference to van Delft and Botermans.]

References

- [1] L. Beltrami. *1512-1532: materiale di studio*, Milano, Tip. U. Allegretti, 1911.
- [2] B. Berenson. "North Italian Painters of the Renaissance", London: Forgotten Books pp. 251-2 (Original work published 1907), 2013.
- [3] G. Cardano. *Opera omnia*, 1663.
- [4] L. Daiken. *Children's games throughout the year*, London: Batsford, 1949.
- [5] V. Huber. "The Yawning Mouth", *Gibecière*, 1, 13-46, 2005.
- [6] J. Ozanam. *Recreations Mathematiques et Physiques...* (2 vols.), Jombert, Paris, 1694.¹
- [7] P. Johnson. *Romano-British Mosaics*, Shire, 2nd edition, 2008.
- [8] H.P. L'Orange, P. J. Nordhagen. *Mosaics: From Antiquity to the Early Middle Ages*, London: Methuen, 1966.
- [9] J. Prévost, *Clever & Pleasant Inventions, Part One*, 1584.
- [10] M. Rule. *Floor Mosaics in Roman Britain*, Macmillan, 1st Edition, 1974.
- [11] T. Sammartini. *The Decorative Floors of Venice*, Merrell Publishers Ltd, 2000.
- [12] D. Schwenter. *Deliciae Physico-mathematicae oder Mathematische und Philosophische Erquickstunden*, 1686.
- [13] J. Shearman. *The Early Italian Pictures in the Collection of Her Majesty The Queen*, CUP, 1983
- [14] J. J. Wecker. *De secretis libri*, 1582.
- [15] S. Witgeest. *Het Natuurlijk Toverboek*, 1686.
- [16] D. Wolfe and T. Rogers. *Puzzlers' Tribute, a Feast for the Mind*, A.K. Peters, 2002.
- [17] <http://www.uriland.it/matematica/DeViribus/Presentazione.html>

¹Numerous editions then appeared in Paris and Amsterdam, some in one volume; About 1723, the work was revised into 4 volumes, sometimes described as 3 volumes and a supplement, published by Claude Jombert, Paris, 1723. "The editor is said to be one Grandin.". In 1778, Jean Étienne Montucla revised this, under the pseudonym M. de C. G. F. [i.e. M. de Chanla, géomètre forézien], published by Claude Antoine Jombert, fils aîné, Paris, 1778, 4 volumes. The author's correct initials appear in the 1790 reissue] This is a considerable reworking of the earlier versions. In particular, the interesting material on conjuring and mechanical puzzles in Vol. IV has been omitted. The bibliography of Ozanam's book is complicated. I have prepared a detailed 7 pp. version covering the 19 (or 20) French and 10 English editions, from 1694 to 1854, as well as 15 related versions - this is part of my *The Bibliography of Some Recreational Mathematics Books*.